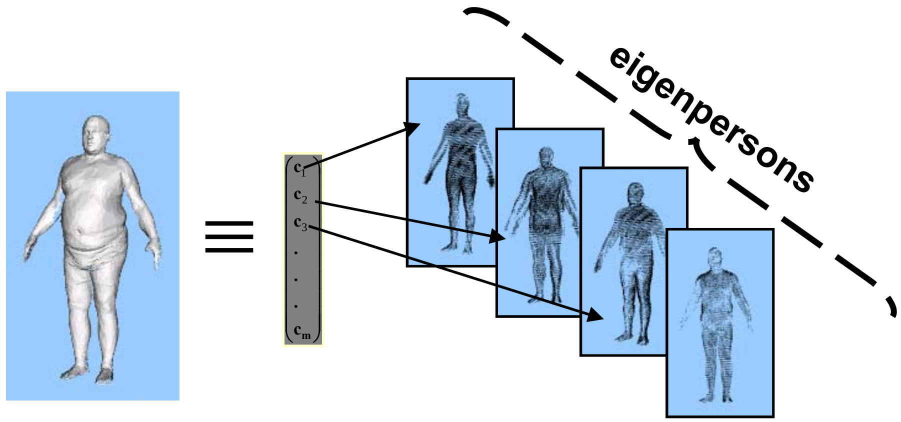
Statistical Shape Analysis



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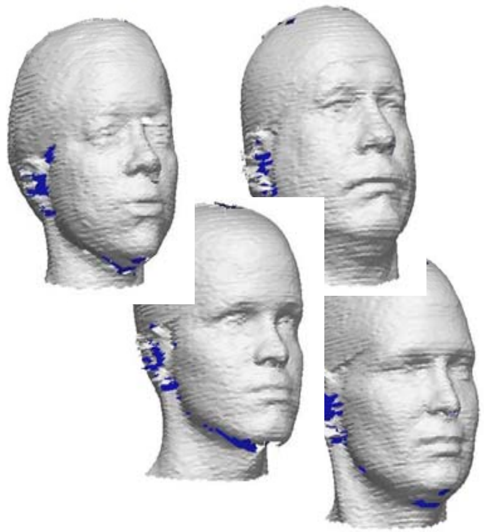
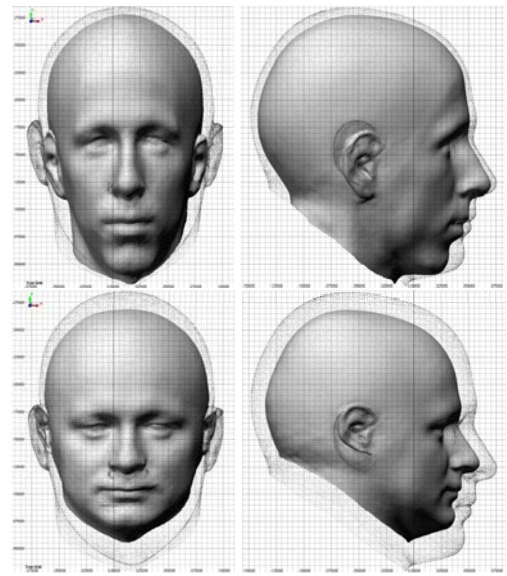
Building statistical models



Class-specific data



Shape variability

Applications:

Recognition	Reconstruction	Pose determination
Segmentation	Motion analysis	Ergonomic design

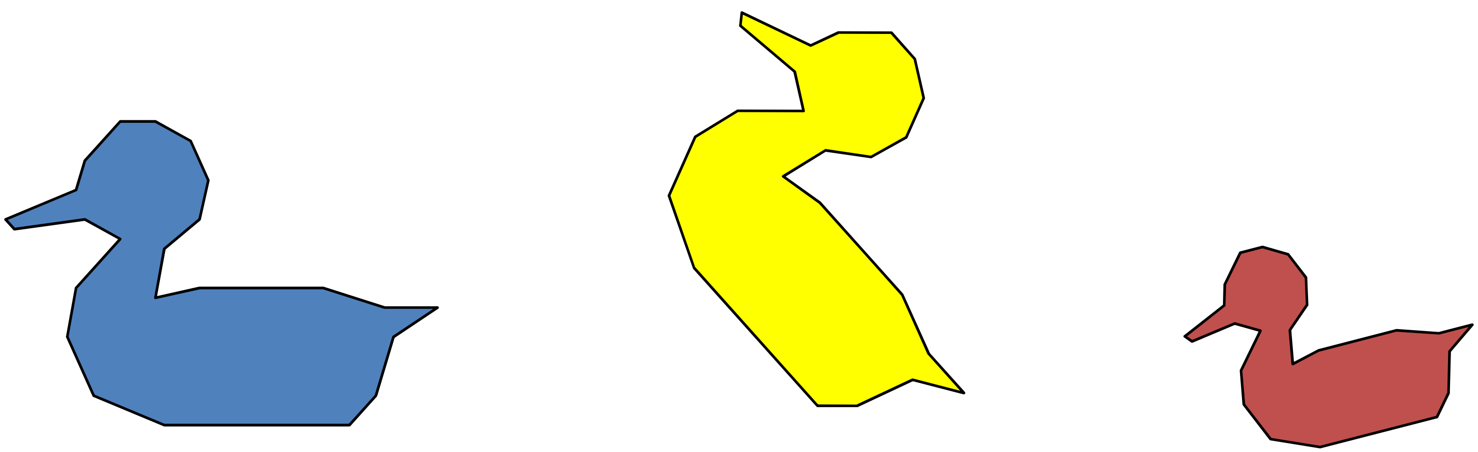
What is SHAPE?

- In everyday language, we often describe shape by analogy
- For example, “Italy is boot-shaped”
- Many applications demand a more rigorous and quantitative account of shape

Shape definition

Shape is all the geometric information that remains when location, scale, and rotational effects are filtered out from an object.

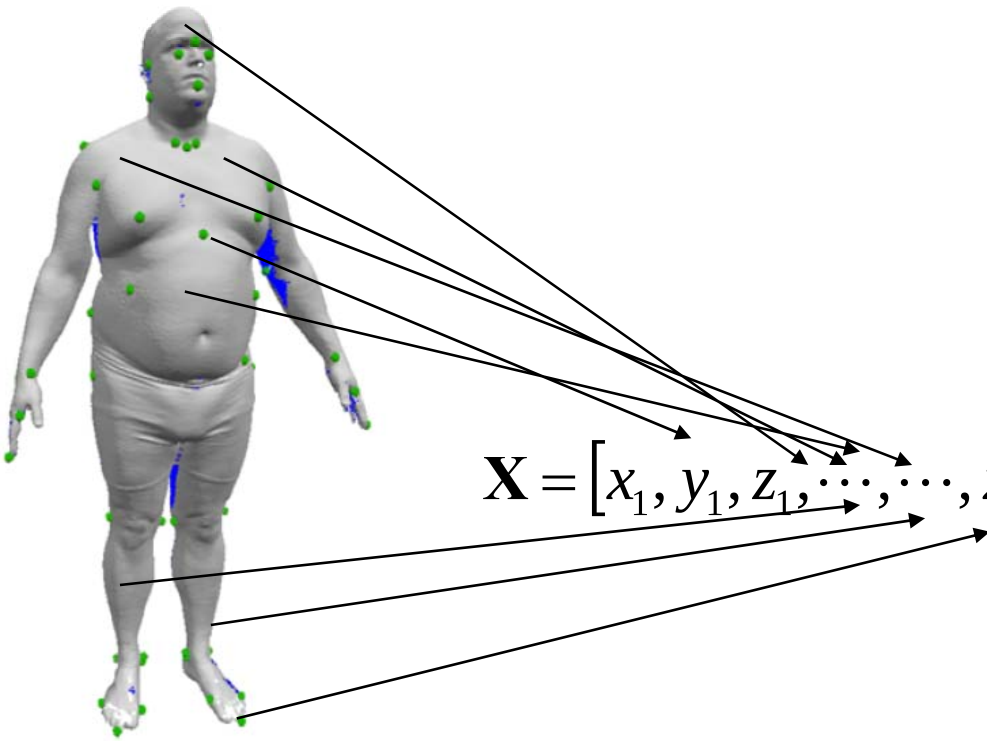
(Kendall 1977)



Statistical Shape Analysis

- A theory of shape
- Statistics + Geometry
- Also called *Geometric Morphometrics* in biological sciences
- Studies patterns of change in shape
- Essential part of many techniques in computer vision: active shape model, morphable model, eigenface, etc.

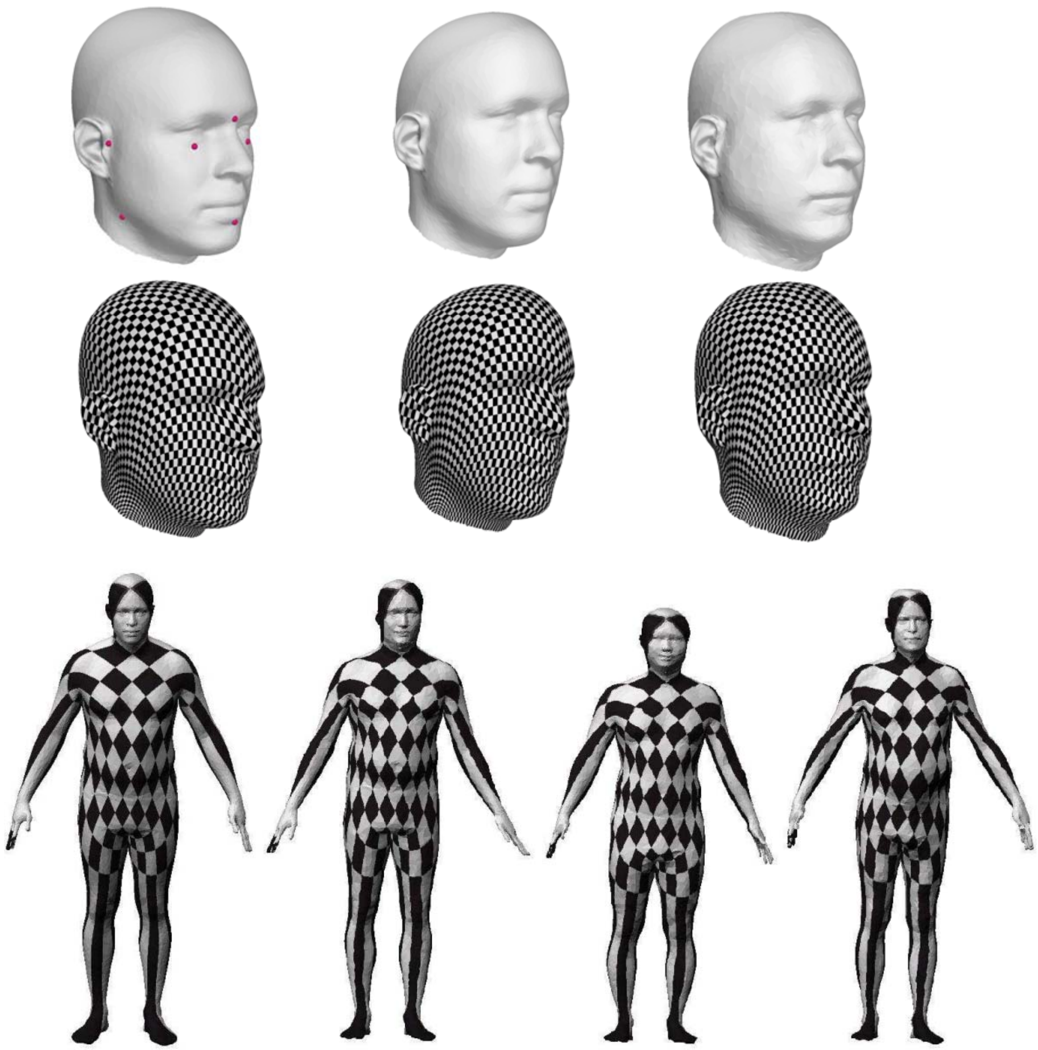
Shape measurement: dense surface data



$$\mathbf{X} = \left[x_1, y_1, z_1, \cdots, \cdots, x_k, y_k, z_k\right]^T$$

$k \approx 100{,}000 - 300{,}000$

Data parameterization



Statistics of shape

- Mean shape
- Variations of shape
- Shape deformation
- Dimensionality reduction
- Factor analysis

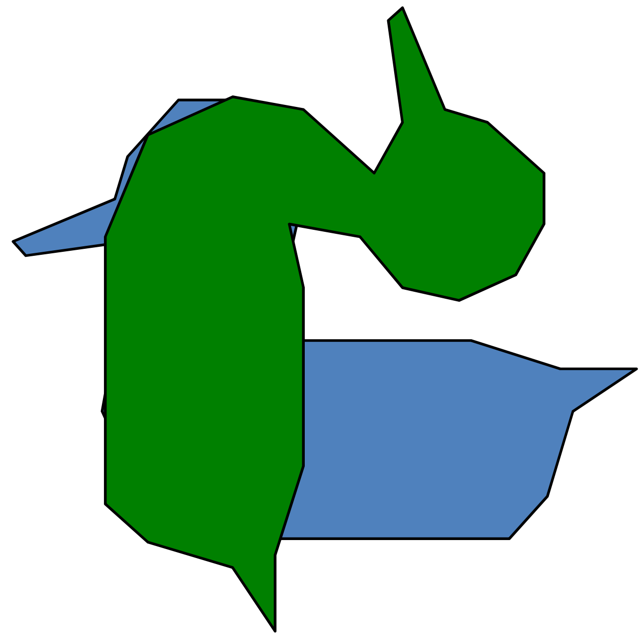
Centring

$$\mathbf{x}_i \leftarrow \mathbf{x}_i - \frac{1}{n}\sum \mathbf{x}_i$$

Removing rotation

Data matrix

$$\mathbf{Z} = \begin{bmatrix} x_1 & y_1 & z_1 \\ \vdots & \vdots & \vdots \\ x_k & y_k & z_k \end{bmatrix}$$

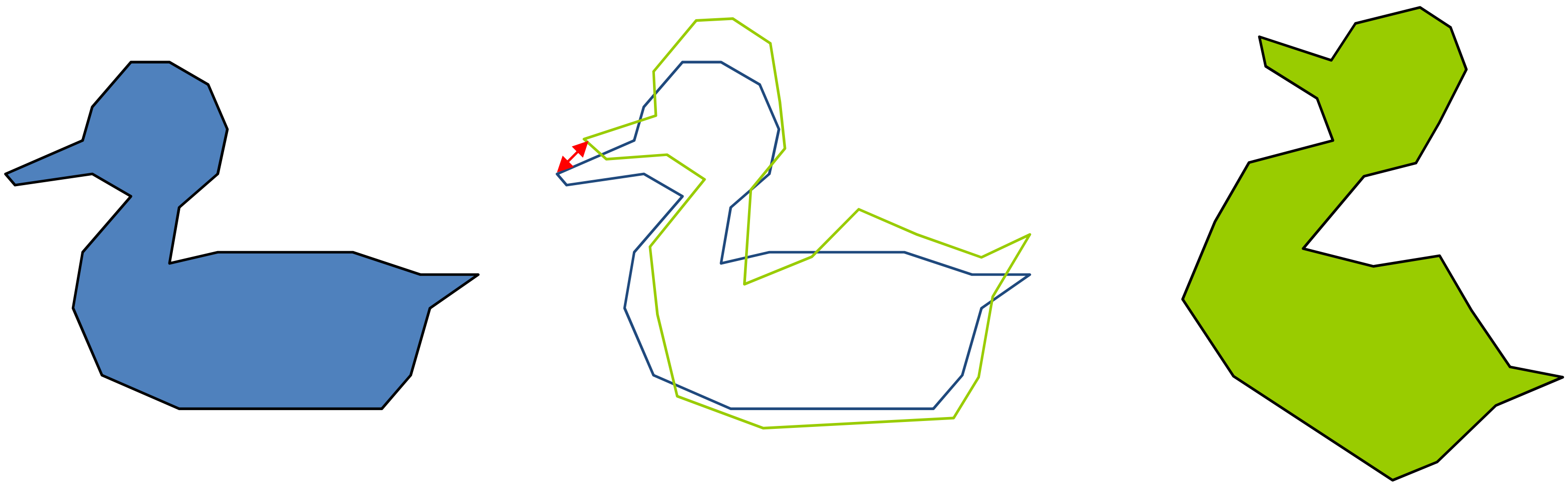


Singular value decomposition (SVD)

$$\mathbf{Z}_1^{\mathrm{T}} \mathbf{Z}_2 = \mathbf{UDV}^{\mathrm{T}}$$

Rotation matrix $\boldsymbol{\Gamma} = \mathbf{UV}^{\mathrm{T}}$

Distance between two shapes



Procrustes distance

$$d(\mathbf{Z}_1, \mathbf{Z}_2) = \left\| \mathbf{Z}_2 - \beta \mathbf{Z}_1 \mathbf{\Gamma} \right\|$$

β is a scale factor. If we want to know size change, $\beta = 1$

Procrustes Alignment

Align shapes such that the total Procrustes distance is minimal.

Given shapes $\mathbf{X}_1, \ldots, \mathbf{X}_n$, the Procrustes mean shape is

$$\mathbf{X_0} = \arg\min_{\mathbf{X_0}} \sum_{i=1}^{n} d^2(\mathbf{X}_i, \mathbf{X_0})$$

Procrustes alignment algorithm

1. Choose an arbitrary shape as the mean shape $\mathbf{X_0}$
2. Align the rest of the shapes to $\mathbf{X_0}$

$$\mathbf{X}_i^p = \mathbf{X}_i \mathbf{\Gamma}_i$$

3. Set new $\mathbf{X_0}$ to the average of the Procrustes-aligned shapes

$$\mathbf{X_0} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{X}_i^p$$

4. Repeat step 2 and 3 until $\mathbf{X_0}$ is stabilized.

Modeling shape variation

Average shape

$$\overline{\Psi} = \frac{1}{N}\sum_{i=N}^{N}\Psi_i$$

Difference vectors

$$\Phi_i = \Psi_i - \overline{\Psi} \qquad \mathbf{A} = \left[\Phi_1 \Phi_2 \cdots \Phi_N\right]$$

Covariance matrix

$$\boldsymbol{\Sigma} = \frac{1}{N-1}\mathbf{A}\mathbf{A}^{\mathrm{T}} = \frac{1}{N-1}\sum_{i=1}^{N}\Phi_i \Phi_i^T$$

Principal Component Analysis

Eigen analysis

$$\mathbf{\Sigma} = \mathbf{U}\Lambda\mathbf{U}^{\mathbf{T}}$$

Basis transformation

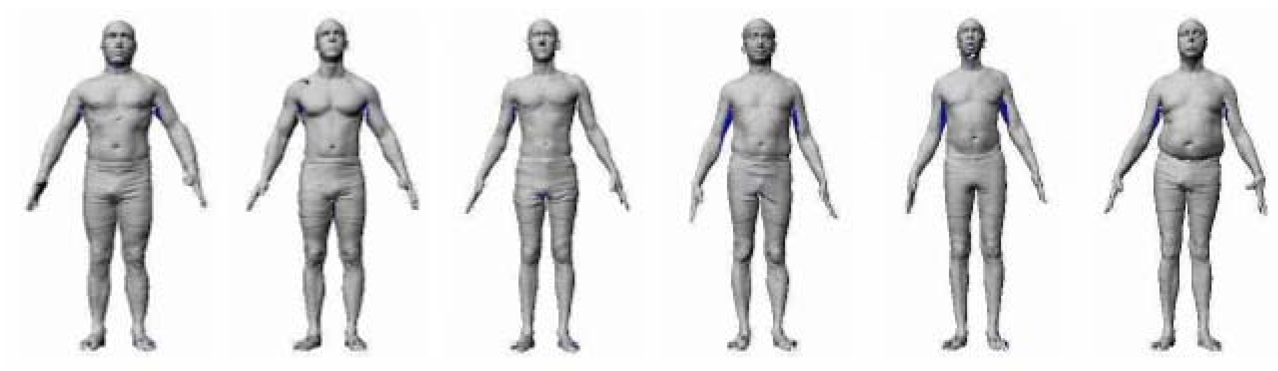
$$\mathbf{b}_i = \mathbf{U}^{\mathbf{T}}\Phi_i$$

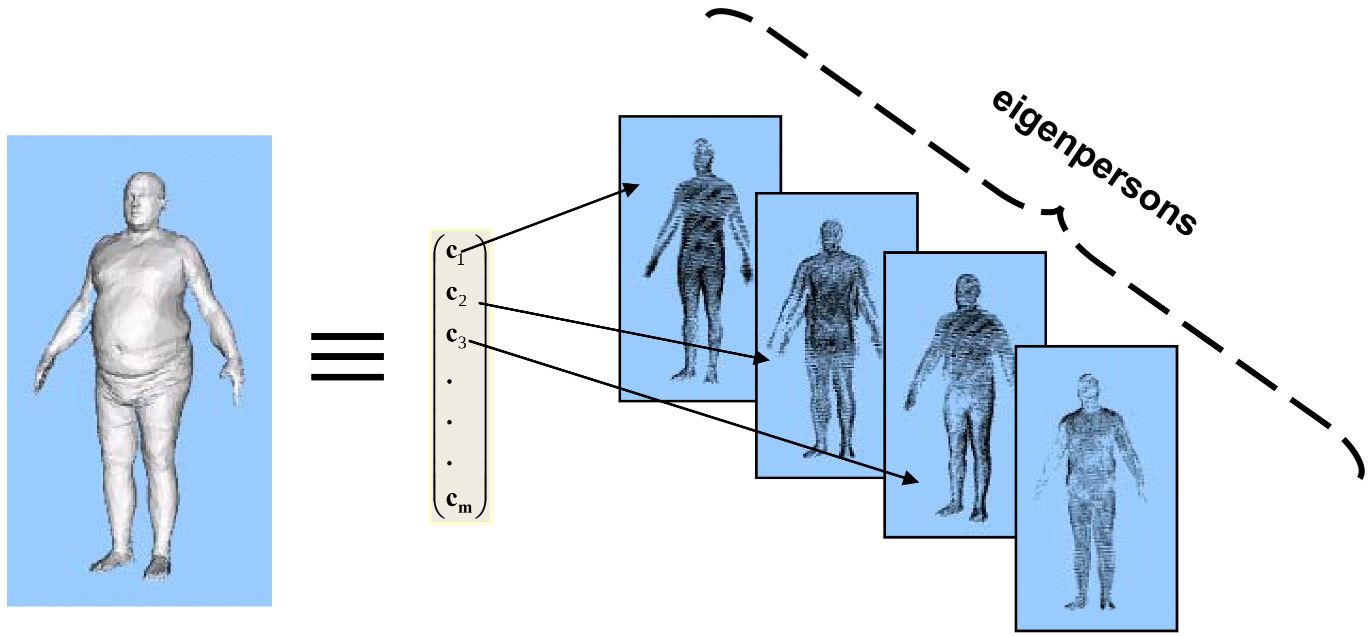
$$\Psi_i = \overline{\Psi} + \mathbf{U}\mathbf{b}_i$$

Dimensionality reduction

$$\Psi_i = \overline{\Psi} + \sum_{j=1}^{M} b_j \mathbf{u}_j \qquad M << N$$

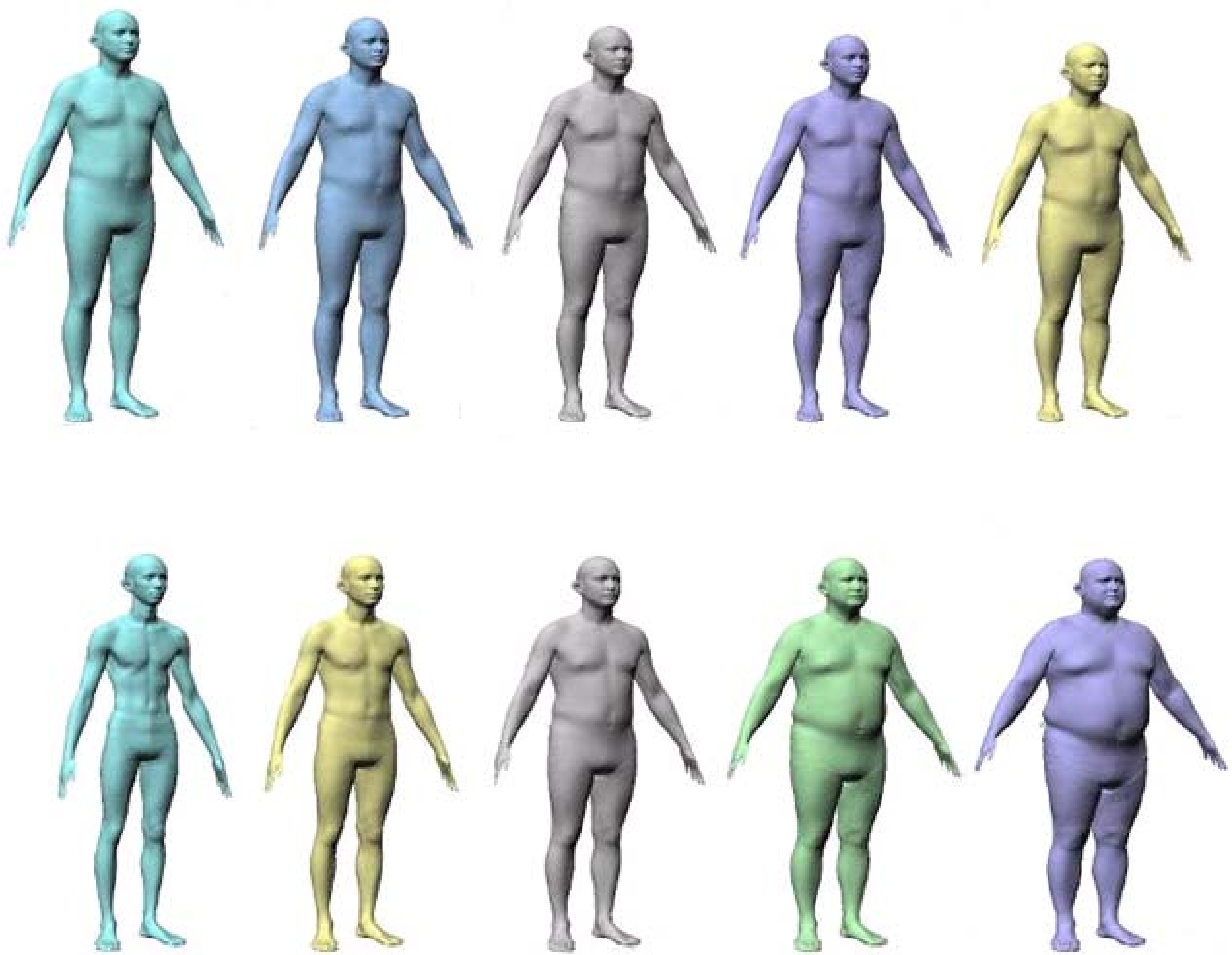
Shape decomposition





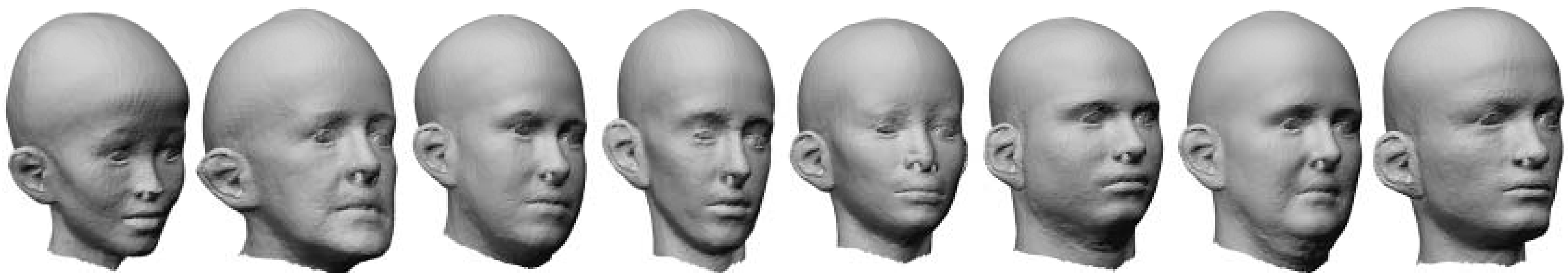
Principal Component Analysis

Main modes of variation



Data from CAESAR dataset

Re-generating a population

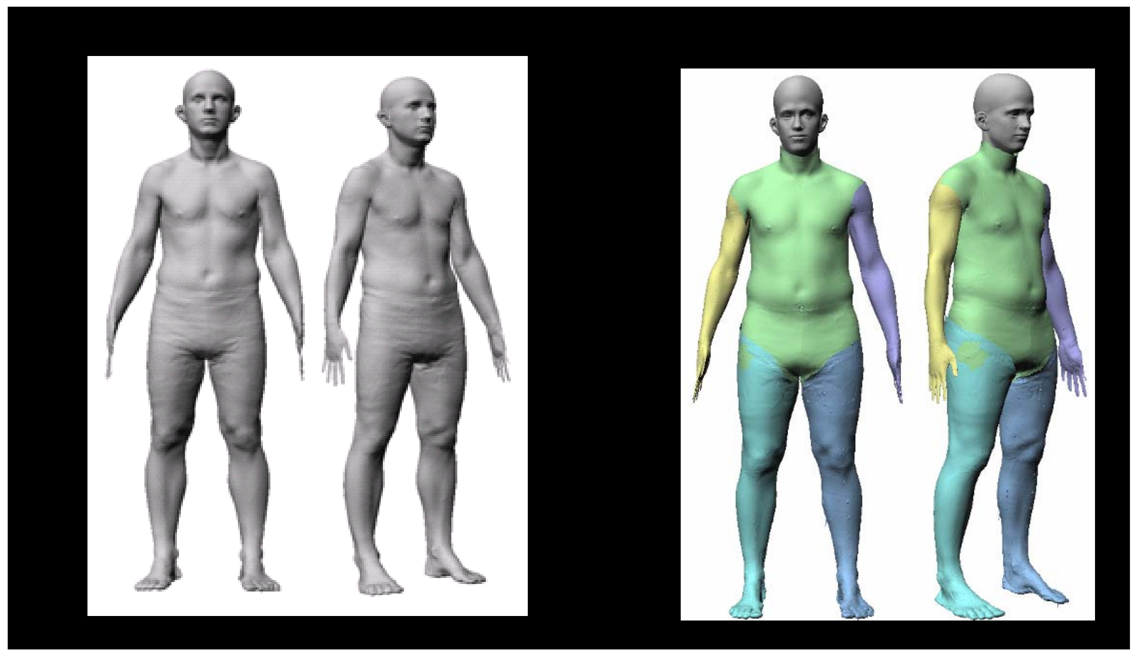


$$\tilde{\Psi} = \overline{\Psi} + \sum_{j=1}^{M} b_j \mathbf{u}_j$$

Potential use in simulation, crowd generation, forensics, etc.

Image-based Reconstruction

Reconstruction from single front-view image



Summary

Statistical shape analysis offers a mathematical quantification of shape

- Applied to dense surface data
  - Captures detailed shape change
  - Intuitive visualization
  - Reveals new patterns of change in shape
- More challenges in geometry processing

References

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