Belief Propagation and its Applications in Computer Vision and Image Processing

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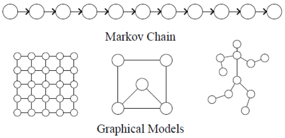
Outline

- Overview
- Markov Random Fields
- Factor Graphs
- Extensions
- Applications

Belief Propagation

- originally by Pearl [Pea88]
- global optimization algorithm for graphical probability models
- exact for tree structured graphs
- approximate for graphs with loops: local optimum over large neighborhood of state space
- continuous and discrete formulations



[SII+03]

Markov Random Fields

- graphical model $G = (V, E)$
- variables represented by nodes
- joint distribution factored into potentials on cliques

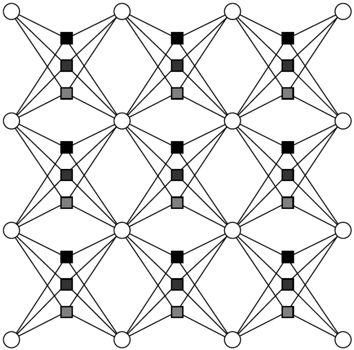
$$P(X) = \prod_{c \in Q} \phi_c(X_c) \tag{1}$$

- Markov property

Factor Graphs

- by Kschischang et al. [KFL01]
- bipartite graph structure
- factor nodes represent potentials, share edges with parameters

$$P(X) = \prod_i f_i(X_{C_i}) \quad (2)$$



[PL08]

BP on MRFs

- messages passed along edges

$$m_{pq}^{t}(x_q) = \int_{x_p} \phi_{pq}(x_p, x_q) \prod_{s \in N(p) \backslash q} m_{sp}^{t-1}(x_p) dx_p \tag{3}$$

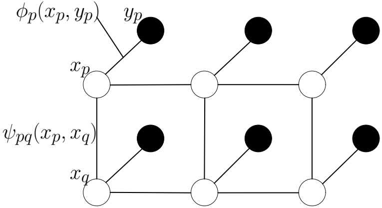
- belief

$$b_p(x_p) = \prod_{q \in N(p)} m_{qp}^{t}(x_p) \tag{4}$$

- exact for tree-structured graphs
- sum-product
- max-product

$$m_{pq}^{t}(x_q) = \max_{x_p} \psi_{pq}(x_p, x_q) \prod_{s \in N(p) \backslash q} m_{sp}^{t-1}(x_p) \tag{5}$$

BP on 2D Grid Pairwise MRFs



typically take joint distribution

$$P(X, Y) = \prod_{(p,q)\in E} \psi_{pq}(x_p, x_q) \prod_{p\in V} \phi_p(x_p, y_p) \tag{6}$$

and write as an energy function

$$E(X, Y) = \sum_{(p,q)\in E} S_{pq}(x_p, x_q) + \sum_{p\in V} D_p(x_p) \tag{7}$$

BP on 2D Grid Pairwise MRFs

max-product becomes min-sum

$$m_{pq}^{t}(x_q) = \min_{x_p}\left(D_p(x_p) + S_{pq}(x_p, x_q) + \sum_{s \in N(p)\backslash q} m_{sp}^{t-1}(x_p)\right) \tag{8}$$

belief becomes

$$b_p(x_p) = \sum_{q \in N(p)} m_{qp}^{t}(x_p) \tag{9}$$

BP on Factor Graphs

- messages passed along edges from variable to factor nodes and vice-versa
- exact for tree-structured graphs
- generalized sum-product algorithm [KFL01]

BP on Factor Graphs

- variable-to-factor message

$$m^t_{p\to f}(x_p) = \prod_{g\in N(p)\backslash f} m^{t-1}_{g\to p}(x_p) \tag{10}$$

- factor-to-variable message

$$m^t_{f\to p}(x_p) = \sum_{N(f)\backslash p} \left(f(X_{N(f)}) \prod_{s\in N(f)\backslash p} m^{t-1}_{s\to f}(x_s) \right) \tag{11}$$

- summary message for variable node

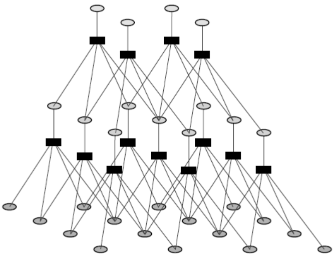
$$b_p(x_p) = \prod_{f\in N(p)} m^t_{f\to p}(x_p) \tag{12}$$

Limitations

- storage and bandwidth requirements
- message updates exponential in clique size
- many message iterations needed for large models
- dimensionality of variables

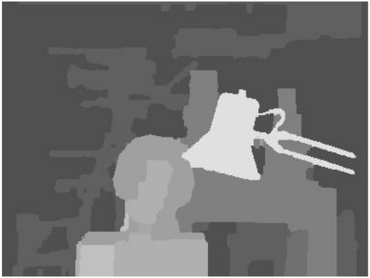
Extensions

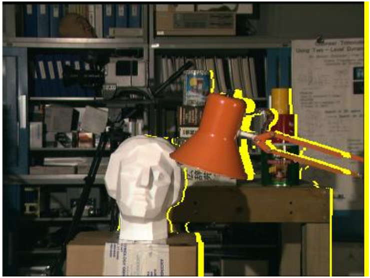
- Hierarchical BP [FH06]
- Generalized BP [YFW03]
- Nonparametric BP [SII+03]
- Linear constraint nodes [PL08]



[PL08]

Stereo





Figures from [SZS03].

Image Restoration

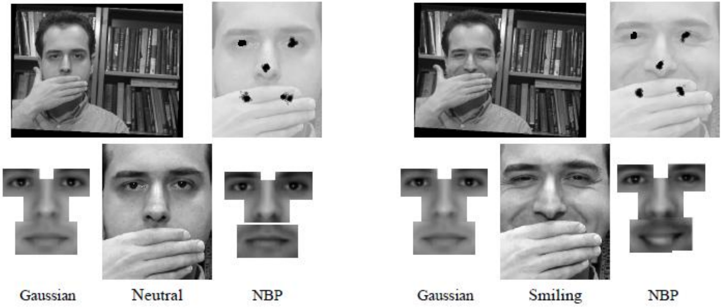

Original


Corrupted


Restoration

Figures from [FH06].

Tracking



Figures from [SII+03].

Thank You

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