CRV 2010 Tutorial Day

Shi-Tomasi, Harris corners and KLT Tracker

Mark Fiala
Ryerson University
Mark.fiala@ryerson.ca

Motivating Interest Points

Finding Correspondences: comparing patches of pixels

- Task: find points between 2 images that correspond to the same object – then use these correspondences for computer vision applications (finding pose, SLAM, building 3D models, locating objects, etc...)

- Single pixel typically not distinctive – use patch of pixels in neighbourhood around a point

- Compare a 2D patch of one image to a patch of the same size in another image – apply some similarity measure (score)

- One similarity measure is SSD (Sum of Squared Differences) – add up the square of differences between pixels in corresponding positions

Finding Correspondences

Compare patches 

$\text{SSD} = \Sigma_i^n (\mathbf{X}_i - \mathbf{Y}_i)^2$

best score = lowest SSD

Selecting Image Patches likely to match

Mark Fiala 2010

Given similarity measures, how can we find corresponding image points?

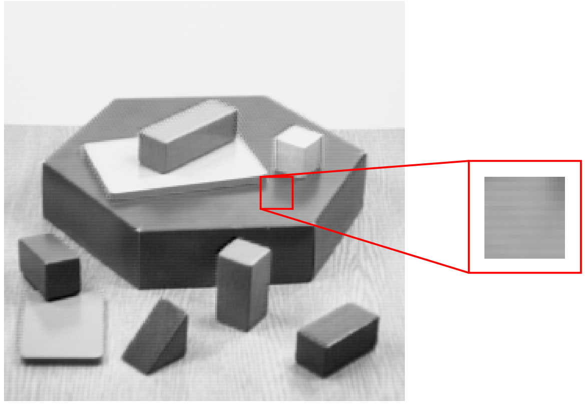
1. Brute force test every possible patch in first image with every possible location in the other image

- Prohibitive computational cost.
- Also, most patches are on edges or blank regions who aren't finding reliable matches anyways

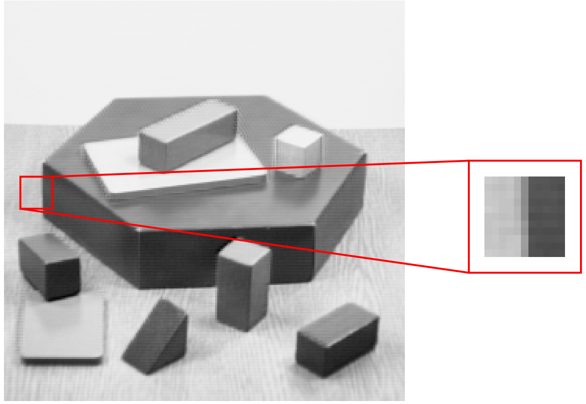
2. Use an interest point detector or corner detector to find a few hundred candidates – just match those

- How can we figure out if a patch is likely to have a unique match in the other image? We can examine a patch first, and declare it an interest point.
- We could test each image patch within its own image first before comparing it with the other image
- See if a patch matches a neighbourhood of points around it. If there is no good match nearby then it is a distinctive patch – label it an interest point. We reduce computational cost by **wh**.
- Is there an even better way to do this, to save on patch comparisons around each point?

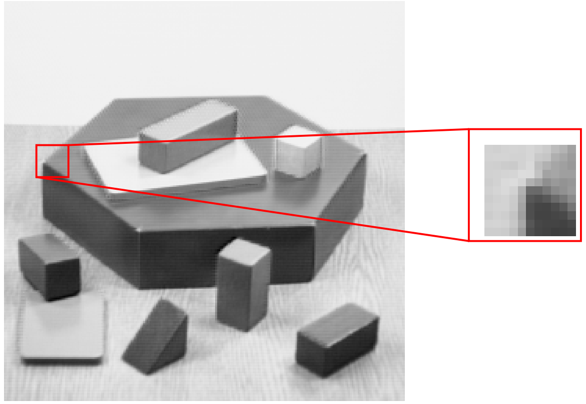
Types of Patches



- Blank region matches many spots
- 2-D uncertainty in matching
- Will match with itself well in close neighbourhood in all directions – and thus will not match uniquely in other image



- Ambiguous matches along edge
- 1-D uncertainty in matching
- Will match with itself well in close neighbourhood along the edge – and thus will not match uniquely in other image



- Distinct region (corner), not ambiguous
- Will NOT match with itself well in close neighbourhood in any directions – and thus could match uniquely in other image

Finding Patches that don't match their Neighbours

- See if a patch matches a neighbourhood of points around it. If there is no good match nearby then it is a distinctive patch – label it an interest point.

- Is there an even better way to do this, to save on patch comparisons around each point?

- Look at spatial derivatives dI/d_x and dI/d_y
- use first order assumption that each pixel will change by $dI/d_x \, \delta x + dI/d_y \, \delta y$
- Find SSD patch comparison as a function of small change $[\delta x, \delta y]^t$

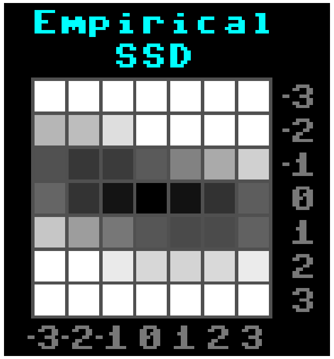
- SSD ~= $||D|| = D^tD$ where D= Assume difference in a pixel is defined by linear approximation – use first derivative X displacement vector

$$\mathbf{D} = \begin{bmatrix} dI_0/d_x & dI_0/d_y \\ dI_1/d_x & dI_1/d_y \\ dI_2/d_x & dI_2/d_y \\ dI_3/d_x & dI_3/d_y \\ \ldots & \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$$

$$\mathbf{SSD} = [\delta x \quad \delta y] \begin{bmatrix} (\Sigma_i dI_i/d_x)^2 & (\Sigma_i dI_i/d_x)\,(\Sigma_i dI_i/d_y) \\ (\Sigma_i dI_i/d_y)\,(\Sigma_i dI_i/d_x)^2 & (\Sigma_i dI_i/d_y)^2 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$$

Finding Patches that don’t match their Neighbours

- correlate patch with patches from same source image
- if a patch matches its neighbours well, it likely won’t be uniquely found in other image
- one way – brute force compare patch with neighbours
- needs b^2p^2 pixel operations – with p=11, b=11 this is ~10^4 operations per pixel

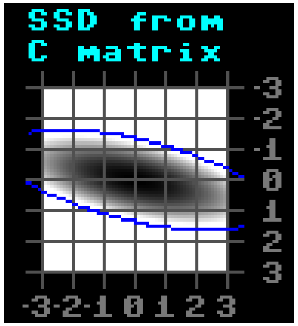


- Can we reduce these operations?
- Approximate SSD using linear assumption of constant spatial derivatives
- Create corner matrix using dI/d_x and dI/d_y
- find SSD using equation $SSD = [x \;\; y] \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

  - Corner matrix C

$$C = \begin{bmatrix} \sum I_{x_i}^2 & \sum I_{x_i} I_{y_i} \\ \sum I_{x_i} I_{y_i} & \sum I_{y_i}^2 \end{bmatrix}$$

- Approximate SSD using C

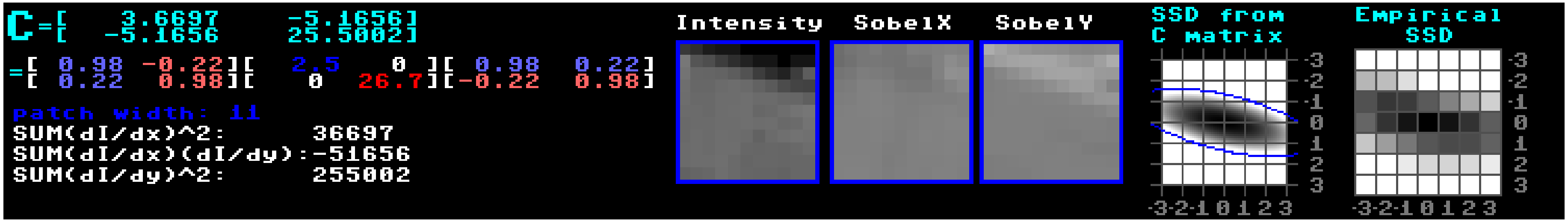


Using C Matrix to find Interest Points

- use klt_corner_gui.exe (can download from http://www.scs.ryerson.ca/~mfiala)
- 2x2 C matrix decomposed to find ellipse major and minor axes
- use minimum of the two axes (smaller eigenvalue of C)
- large minimum eigenvalue = tight ellipse

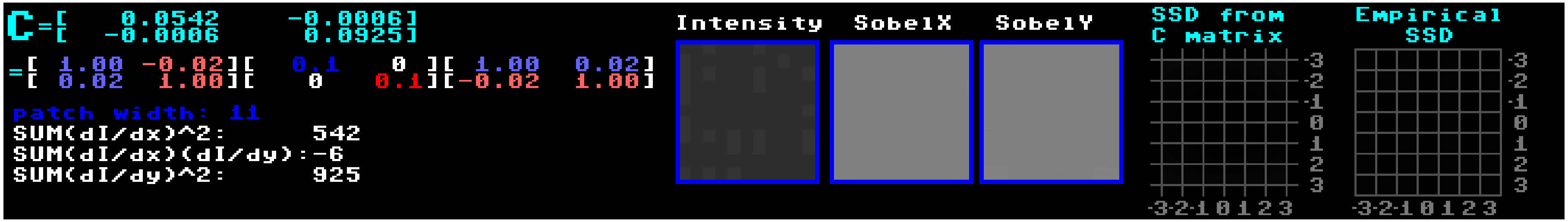
= large change in SSD for small change in position = distinctive point

Edge patch – not so distinct (min eigenvalue=2.5)



```
C=[   3.6697     -5.1656]
  [  -5.1656     25.5002]
=[ 0.98 -0.22][   2.5      0  ][ 0.98   0.22]
 [ 0.22  0.98][    0    26.7][-0.22   0.98]
patch width: 11
SUM(dI/dx)^2:         36697
SUM(dI/dx)(dI/dy):-51656
SUM(dI/dy)^2:         255002
```

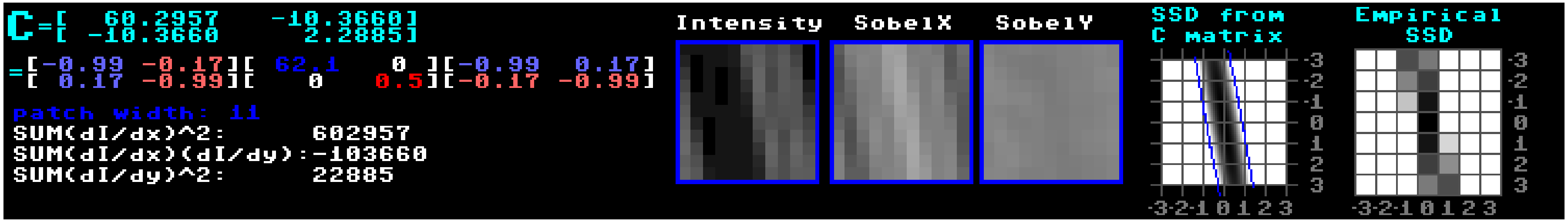
Bland region – no real change in SSD, not distinct at all (min eigenvalue=0.1)



```
C=[   0.0542     -0.0006]
  [  -0.0006      0.0925]
=[ 1.00 -0.02][   0.1      0  ][ 1.00   0.02]
 [ 0.02  1.00][    0     0.1][-0.02   1.00]
patch width: 11
SUM(dI/dx)^2:         542
SUM(dI/dx)(dI/dy):-6
SUM(dI/dy)^2:         925
```

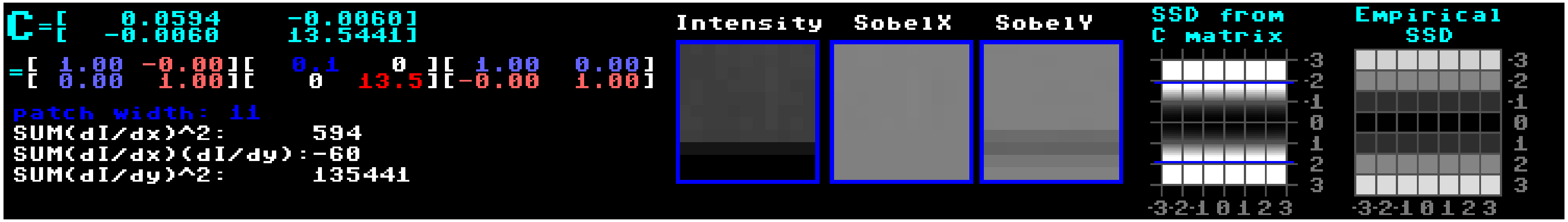
More patches

Edge patch – not so distinct (min eigenvalue=0.5)



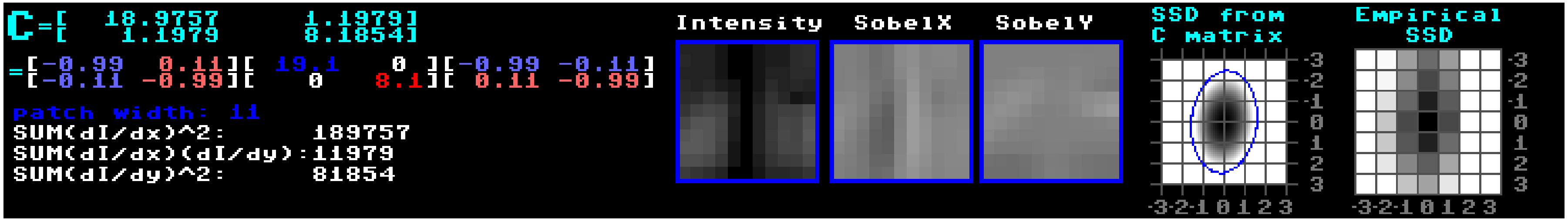
```
C=[  60.2957    -10.3660]
  [ -10.3660      2.2885]
=[-0.99 -0.17][ 62.1     0 ][-0.99  0.17]
 [ 0.17 -0.99][    0    0.5][-0.17 -0.99]
patch width: 11
SUM(dI/dx)^2:          602957
SUM(dI/dx)(dI/dy):-103660
SUM(dI/dy)^2:          22885
```

Edge patch – not so distinct (min eigenvalue=0.1)



```
C=[   0.0594     -0.0060]
  [  -0.0060     13.5441]
=[ 1.00 -0.00][   0.1     0 ][ 1.00   0.00]
 [ 0.00  1.00][    0   13.5][-0.00   1.00]
patch width: 11
SUM(dI/dx)^2:          594
SUM(dI/dx)(dI/dy):-60
SUM(dI/dy)^2:          135441
```

region with edges in both directions, more distinct (min eigenvalue=8.1)

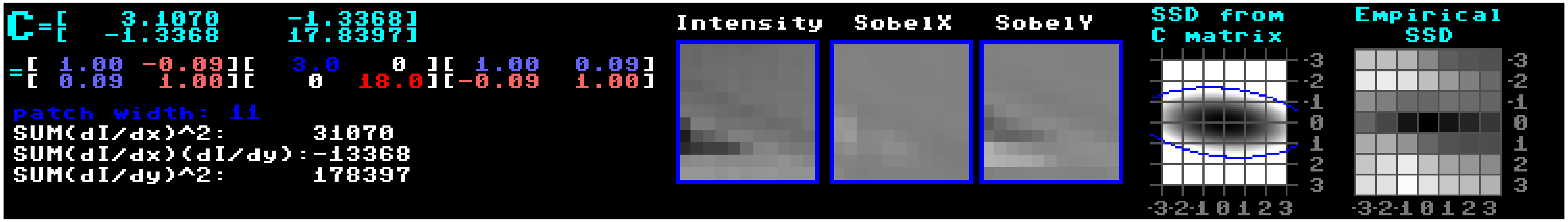


```
C=[  18.9757       1.1979]
  [   1.1979       8.1854]
=[-0.99  0.11][ 19.1     0 ][-0.99 -0.11]
 [-0.11 -0.99][    0    8.1][ 0.11 -0.99]
patch width: 11
SUM(dI/dx)^2:          189757
SUM(dI/dx)(dI/dy):11979
SUM(dI/dy)^2:          81854
```

Mark Fiala 2010

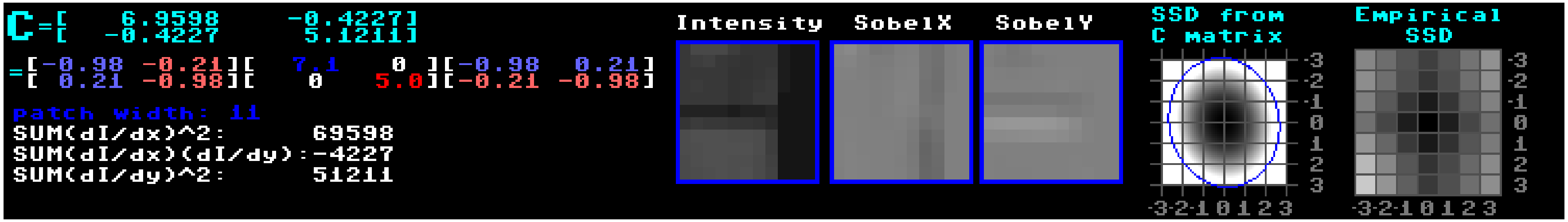
More patches

Edge patch – not so distinct (min eigenvalue=3.0)



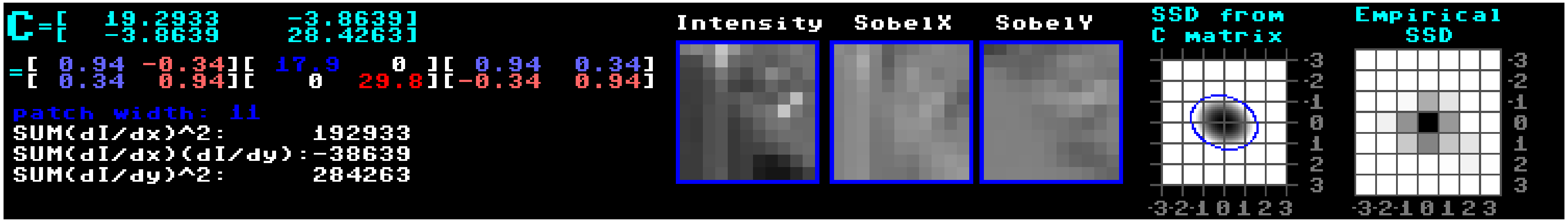
```
C=[   3.1070      -1.3368]
  [  -1.3368      17.8397]
=[ 1.00 -0.09][ 3.0     0  ][ 1.00  0.09]
 [ 0.09  1.00][  0    18.0 ][-0.09  1.00]
patch width: 11
SUM(dI/dx)^2:        31070
SUM(dI/dx)(dI/dy):-13368
SUM(dI/dy)^2:        178397
```

Corner patch – more distinct (min eigenvalue=5.0)



```
C=[   6.9598      -0.4227]
  [  -0.4227       5.1211]
=[-0.98 -0.21][ 7.1     0  ][-0.98  0.21]
 [ 0.21 -0.98][  0     5.0 ][-0.21 -0.98]
patch width: 11
SUM(dI/dx)^2:        69598
SUM(dI/dx)(dI/dy):-4227
SUM(dI/dy)^2:        51211
```

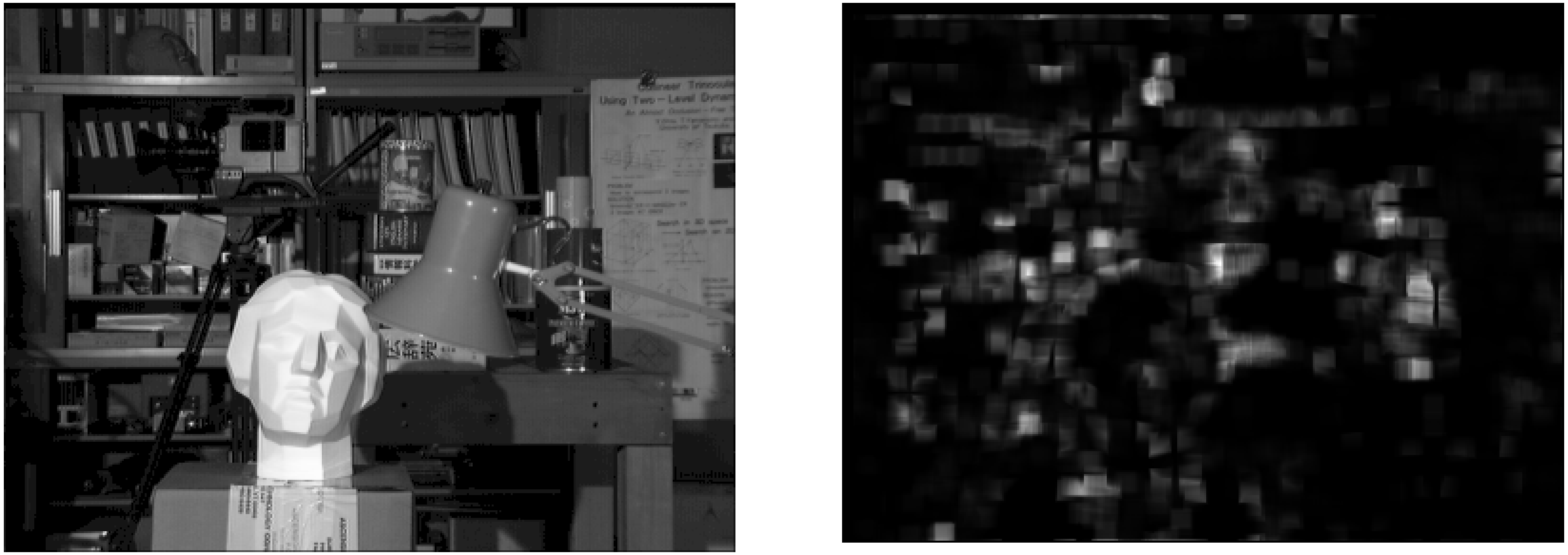
even more distinct (min eigenvalue=19.3)



```
C=[  19.2933      -3.8639]
  [  -3.8639      28.4263]
=[ 0.94 -0.34][ 17.9    0  ][ 0.94  0.34]
 [ 0.34  0.94][  0    29.8 ][-0.34  0.94]
patch width: 11
SUM(dI/dx)^2:        192933
SUM(dI/dx)(dI/dy):-38639
SUM(dI/dy)^2:        284263
```

Min Eigen Image

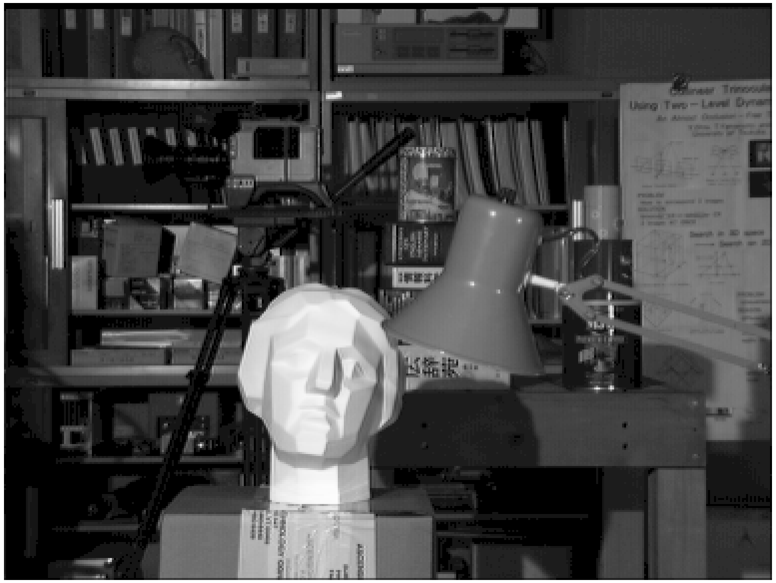
Calculate min eigenvalue for each pixel position

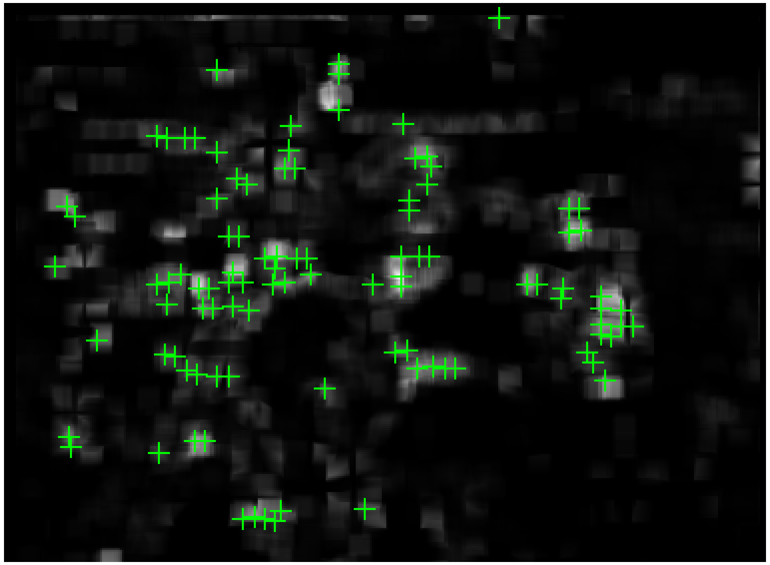


Min Eigen Image

Calculate min eigenvalue for each pixel position

Find local peaks – write these out as interest points





Ways to Speed up Corner Detection

- Finding eigenvalues of corner matrix C requires some calculation
- we can cut some corners if we use the fact that the trace and determinant of the matrix do not change with rotation (U,V matrices from SVD)

```
C=[   19.2933      -3.8639]
  [   -3.8639      28.4263]
=[ 0.94  -0.34][ 17.9     0  ][ 0.94   0.34]
 [ 0.34   0.94][   0    29.8 ][-0.34   0.94]
```

- Label the two eigenvalues (A,B) , trace = A+B, determinant = A x B
- all we are interested in is if the smaller of A and B is greater than a threshold
- Harris corner detector uses metric

$$\det(A) - \kappa \operatorname{trace}^2(A)$$

- suggested k=0.25. Only if this quantity is above a threshold do we calculate the full eigenvalues – saves lots of calculations

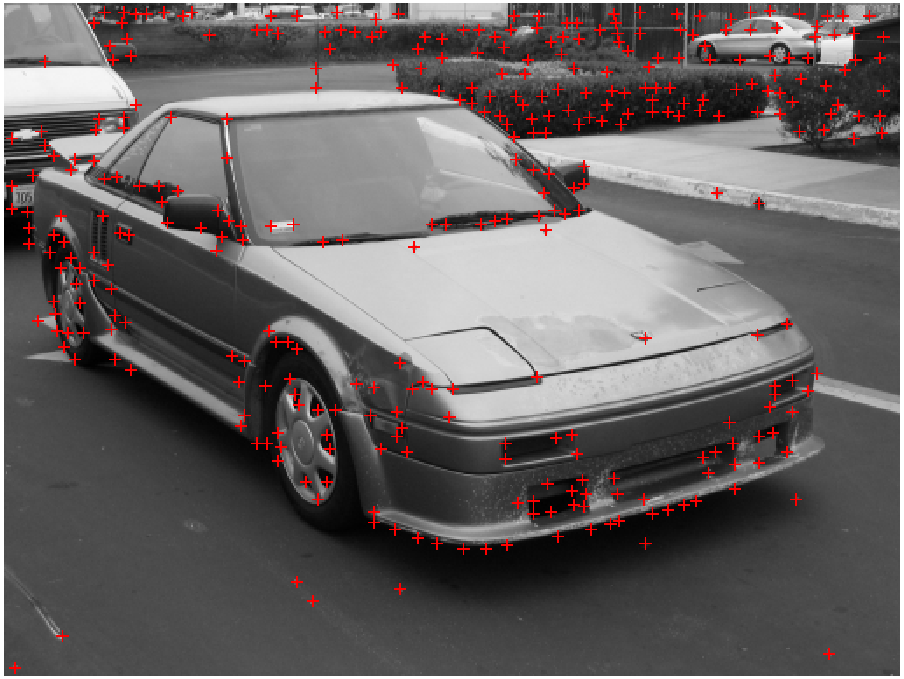
Finding KLT corners – boat example

Finding KLT corners – boat example

Finding KLT corners – car example

Finding KLT corners – car example

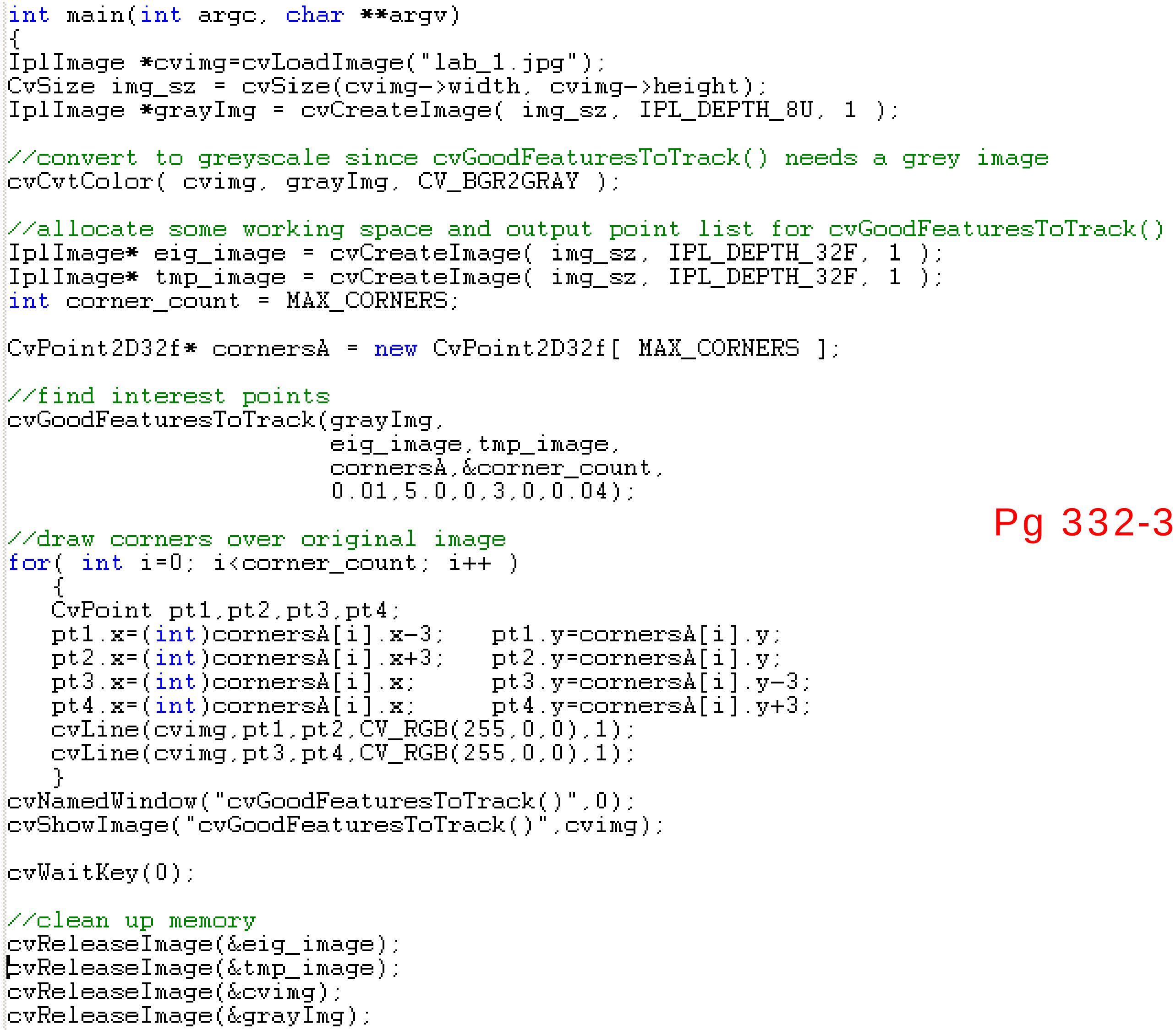
KLT/Harris corners doesn't give good results for all images



OpenCV interest point detector – cvGoodFeaturesToTrack()

Implements C-matrix, min eigenvalue method (Lec 5 KLT/Harris corner detector).

- Needs greyscale IplImage as input, provides CvPoint2D32f list output



```
int main(int argc, char **argv)
{
IplImage *cvimg=cvLoadImage("lab_1.jpg");
CvSize img_sz = cvSize(cvimg->width, cvimg->height);
IplImage *grayImg = cvCreateImage( img_sz, IPL_DEPTH_8U, 1 );

//convert to greyscale since cvGoodFeaturesToTrack() needs a grey image
cvCvtColor( cvimg, grayImg, CV_BGR2GRAY );

//allocate some working space and output point list for cvGoodFeaturesToTrack()
IplImage* eig_image = cvCreateImage( img_sz, IPL_DEPTH_32F, 1 );
IplImage* tmp_image = cvCreateImage( img_sz, IPL_DEPTH_32F, 1 );
int corner_count = MAX_CORNERS;

CvPoint2D32f* cornersA = new CvPoint2D32f[ MAX_CORNERS ];

//find interest points
cvGoodFeaturesToTrack(grayImg,
                      eig_image,tmp_image,
                      cornersA,&corner_count,
                      0.01,5.0,0,3,0,0.04);

//draw corners over original image
for( int i=0; i<corner_count; i++ )
   {
   CvPoint pt1,pt2,pt3,pt4;
   pt1.x=(int)cornersA[i].x-3;   pt1.y=cornersA[i].y;
   pt2.x=(int)cornersA[i].x+3;   pt2.y=cornersA[i].y;
   pt3.x=(int)cornersA[i].x;     pt3.y=cornersA[i].y-3;
   pt4.x=(int)cornersA[i].x;     pt4.y=cornersA[i].y+3;
   cvLine(cvimg,pt1,pt2,CV_RGB(255,0,0),1);
   cvLine(cvimg,pt3,pt4,CV_RGB(255,0,0),1);
   }
cvNamedWindow("cvGoodFeaturesToTrack()",0);
cvShowImage("cvGoodFeaturesToTrack()",cvimg);

cvWaitKey(0);

//clean up memory
cvReleaseImage(&eig_image);
cvReleaseImage(&tmp_image);
cvReleaseImage(&cvimg);
cvReleaseImage(&grayImg);
```

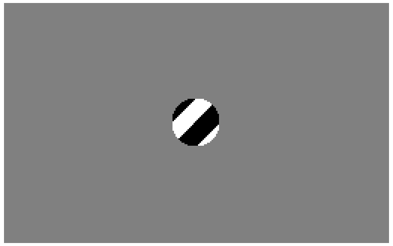
Pg 332-334

Lec11_files.zip - cvGoodFeaturesToTrack_image.cpp

KLT tracker

Aperture Problem: a small pixel neighbourhood can only detect motion perpendicular to edge

-therefore each pixel position can only constrain optic flow **V** to a 1D space.



Optic Flow Equation: $\nabla I^T \cdot \vec{V} = -I_t$

Use optic flow equation for each pixel in patch – use least squares fit to find V

$$\begin{aligned}
I_{x_1}V_x + I_{y_1}V_y &= -I_{t_1}\\
I_{x_2}V_x + I_{y_2}V_y &= -I_{t_2}\\
&\vdots\\
I_{x_n}V_x + I_{y_n}V_y &= -I_{t_n}
\end{aligned}$$

KLT tracker

Optic Flow Equation: $\nabla I^T \cdot \vec{V} = -I_t$

Use optic flow equation for each pixel in patch – use least squares fit to find V

$$\begin{array}{l} I_{x1} V_x + I_{y1} V_y = -I_{t1} \\ I_{x2} V_x + I_{y2} V_y = -I_{t2} \\ \vdots \\ I_{xn} V_x + I_{yn} V_y = -I_{tn} \end{array}$$

Use optic flow equation for each pixel in patch – use least squares fit to find V

$$\begin{bmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \vdots \\ I_{xn} & I_{yn} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} -I_{t1} \\ -I_{t2} \\ \vdots \\ -I_{tn} \end{bmatrix}$$

This is of the form Ax=B. Least squares solution is $x = (A^t A)^{-1} A^t B$

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \sum I_{x_i}^2 & \sum I_{x_i} I_{y_i} \\ \sum I_{x_i} I_{y_i} & \sum I_{y_i}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum I_{x_i} I_{t_i} \\ -\sum I_{y_i} I_{t_i} \end{bmatrix}$$

Notice left quantity is inverse of **C** matrix used in corner detection.

KLT tracker

Some links:

http://en.wikipedia.org/wiki/Optical_flow

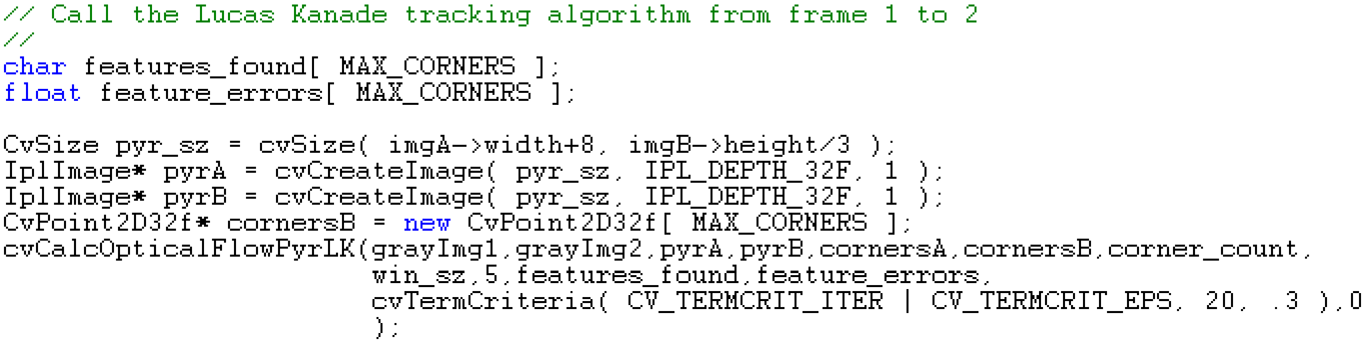
http://en.wikipedia.org/wiki/Lucas%E2%80%93Kanade_Optical_Flow_Method

OpenCV KLT tracker – cvCalcOpticalFlowPyrLK()

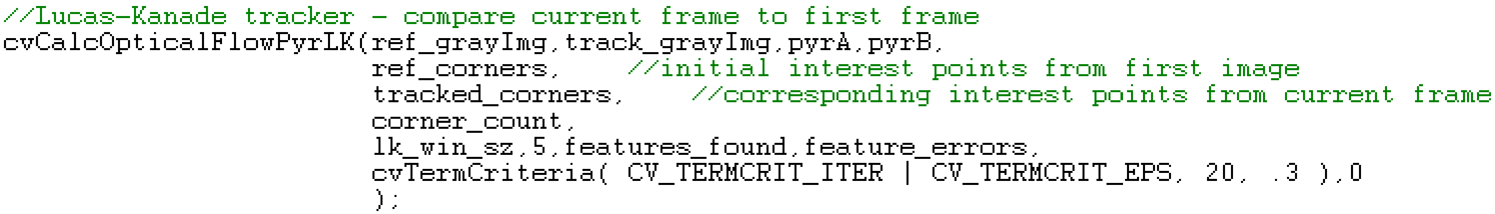
Pg 332-334

Implements KLT tracking using C^{-1} matrix (Lecture 6)

- Needs start points – uses output of cvGoodFeaturesToTrack()
- Iterates a few times for each point (each step gives linear movement)
- Processes on multiple levels (image pyramid)
- Image pyramid for each (greyscale) image must be created first (pyramid consists of a set of images of different size)



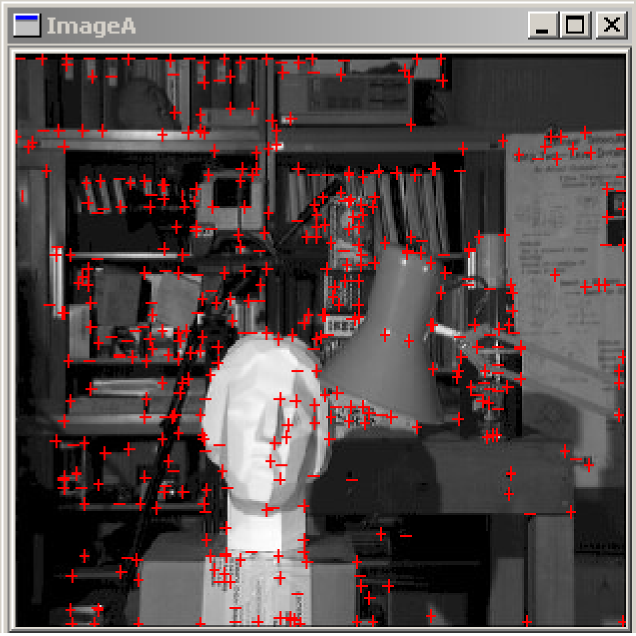
```
// Call the Lucas Kanade tracking algorithm from frame 1 to 2
//
char features_found[ MAX_CORNERS ];
float feature_errors[ MAX_CORNERS ];

CvSize pyr_sz = cvSize( imgA->width+8, imgB->height/3 );
IplImage* pyrA = cvCreateImage( pyr_sz, IPL_DEPTH_32F, 1 );
IplImage* pyrB = cvCreateImage( pyr_sz, IPL_DEPTH_32F, 1 );
CvPoint2D32f* cornersB = new CvPoint2D32f[ MAX_CORNERS ];
cvCalcOpticalFlowPyrLK(grayImg1,grayImg2,pyrA,pyrB,cornersA,cornersB,corner_count,
                       win_sz,5,features_found,feature_errors,
                       cvTermCriteria( CV_TERMCRIT_ITER | CV_TERMCRIT_EPS, 20, .3 ),0
                       );
```

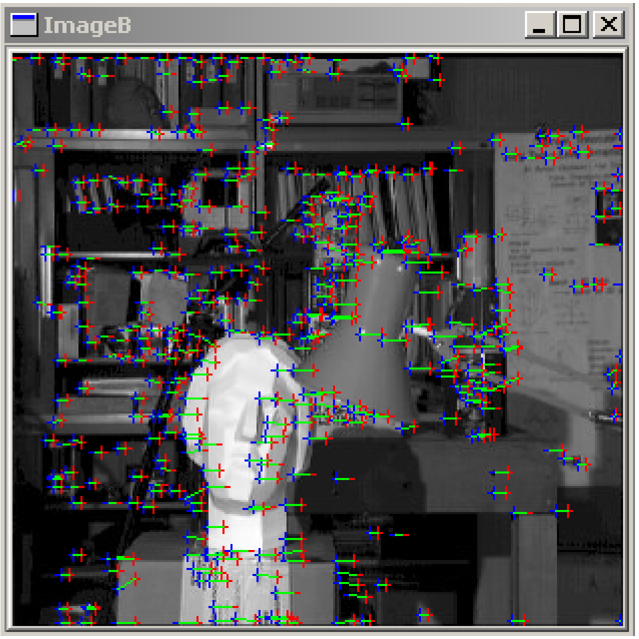


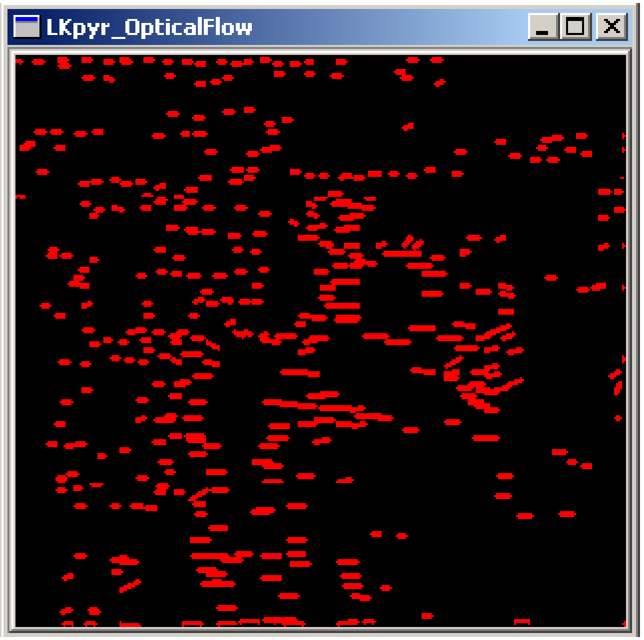
```
//Lucas-Kanade tracker - compare current frame to first frame
cvCalcOpticalFlowPyrLK(ref_grayImg,track_grayImg,pyrA,pyrB,
                       ref_corners,    //initial interest points from first image
                       tracked_corners,    //corresponding interest points from current frame
                       corner_count,
                       lk_win_sz,5,features_found,feature_errors,
                       cvTermCriteria( CV_TERMCRIT_ITER | CV_TERMCRIT_EPS, 20, .3 ),0
                       );
```

Lec11_files.zip - cvCalcOpticalFlowPyrLK.cpp

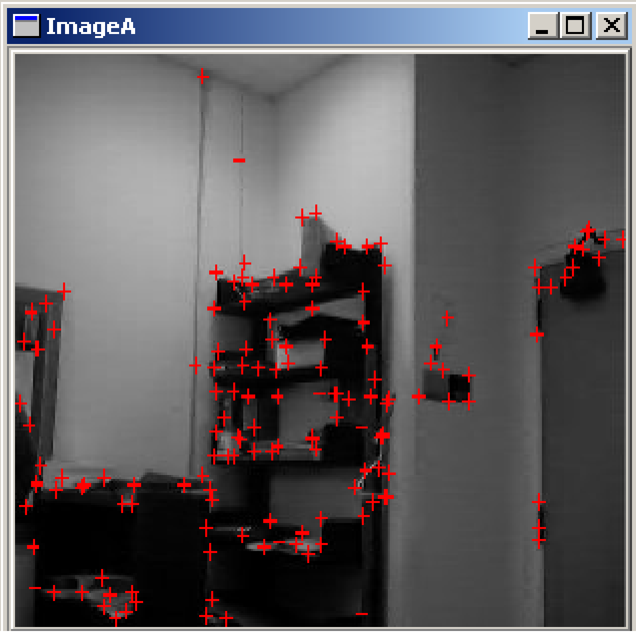
OpenCV KLT tracker – Lab1,2 example

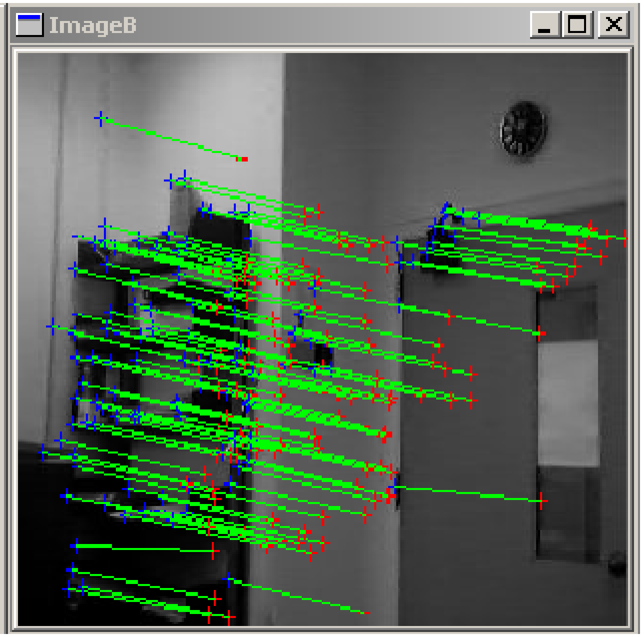


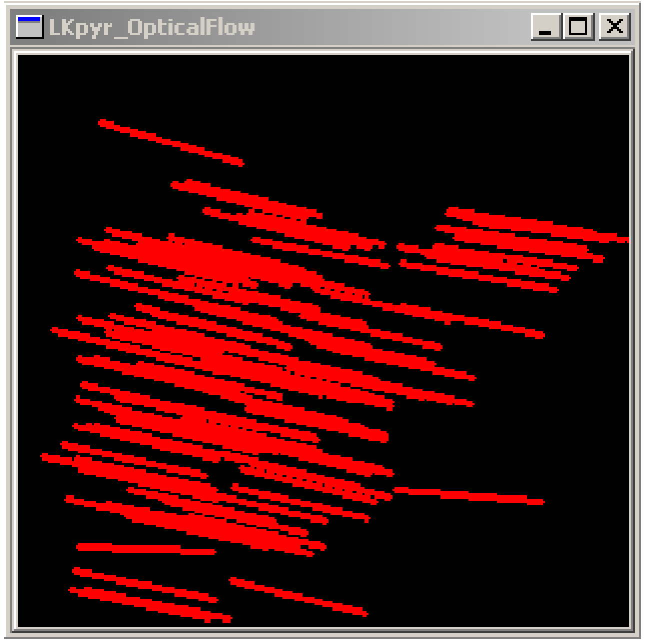




OpenCV KLT tracker – another example







Mark Fiala 2010