CRV 2010 Tutorial Day

2-View Geometry

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3-Vectors for image points and lines

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2D Homogeneous Points

- Add 3rd number to a 2D point on image plane (add 1)
- Extend $[u,v]^t$ to $[u,v,1]^t$
- 2D point is now represented by 3D line, can multiply entire vector by a constant and it's still the same point

$$k[u,v,1]^t = [ku,kv,k]^t = [u,v,1]^t$$

2D Projective Geometry

- Point is represented by 3D line passing through origin
  (2D point -> 3D vector)
  $p=k[u,v,1]^t$
- Image plane is horizontal plane at z=1
- Image point is where 3D line passes through image plane
- Image line represented by where 3D plane passes through image plane
- 3D plane passes through origin, defined by normal (perpendicular) 3D vector $L=[a,b,c]^t$
- Standard line definition: $ax+by+c=0$
- Line defined as vectors perpendicular to vector L (dot product=0)
  $L^t p=0$

3-Vectors – scale does not matter

- Image point: $[u,v,1]^t$
- Image line: $[a,b,c]^t$

Homography and Fundamental Matrices

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Two View Geometry

- When you have two images, what relations can you find between them?
- A homography matrix maps a point in one image to a point in another, but only under special cases:
  1-both looking at the same plane (any R or T allowed) – can do planar mosaic
  2-only rotation R, no translation T –can do panoramic mosaic
- Is there anything we can do for two arbitrary images – with unknown R & T that are not aimed at a plane?

Point->Line Mapping

- With any configuration with T!=0 we can find a point-to-line mapping
- A point in one image maps to a line in another image
- For a single camera, all points along a line in 3D space fall onto a single image point. Project all those points onto another camera, they appear as a line emanating from the image of the focal point of the first camera.
- Imagine a laser beam "shooting" out of Cam_1, see the laser beam in second camera Cam_2 –will appear as a line.
- Therefore a point p_1 in Cam_1 maps to a line l_2 in Cam_2
- What is the relation?
  Answer $l_2 = Fp_1$ **F** is the fundamental matrix (3x3)
- Since $l^t p = p^t l = 0$ for points p lying on line l, we can say an object at p_1 in Cam_1 must lie upon l_2 Cam_2 –therefore $p_2^t F p_1 = 0$

Matrices, World-Image and Image-Image Relationships

Projection Matrix $p=PX$

- Homogeneous point p as a function of world coords X
- Size P=3x4, not invertible (cannot go from image point to world point)

Homography Matrix $p=HX$ or $p_2= \mathbf{H}p_1$

- Homogeneous point p as a function of world planar coords X
- Or rotating but not translating camera, Homogeneous points p_1,p_2
- Point in one image maps to point in other image via H
- Size H=3x3, invertible (can go from image point to world planar point)

Fundamental Matrix $p_2^t\mathbf{F}p_1 =0$

- Line $l_2 = \mathbf{F}p_1$ as a function of homogeneous point p_1
- Size F=3x3, not invertible (cannot go from point back to line, F contains T_x matrix, $\det(T_x)=0$, cannot be inverted)

3-View Tensor $p_3= \mathbf{T}(p_1, p_2)$

- Homogeneous point p_3 in 3rd image as a function of homogeneous point p_1 in first image and p_2 in second image
- Works for arbitrary R and T between cameras, world not restricted to planar case
- Size tensor=3x3x3 (27 numbers)

Entities where scale doesn't matter

- All the following are independent of scale: each is represented by one more number than the degrees of freedom

Homogeneous point (2D point->3D line)

$p=k[u, v, 1]^t$

- 3 numbers, 2 dof (degrees of freedom)

Line

$l=k[a, b, c]^t$

- 3 numbers, 2 dof

Projection Matrix

$p=PX$

- Homogeneous point p as a function of world coords X
- 12 numbers, 11 dof

Homography Matrix

$p=HX$ or $p_2= \mathbf{H}p_1$

- Map a homogeneous point between a plane and image, or between rotated images
- 9 numbers, 8 dof

Fundamental Matrix

$p_2^t\mathbf{F}p_1 =0$

- Line $l_2 = \mathbf{F}p_1$ as a function of homogeneous point p_1
- 9 numbers, 8 dof

3-View Tensor

$p_3= \mathbf{T}(p_1, p_2)$

- Homogeneous point p_3 in 3rd image as a function of homogeneous point p_1 in first image and p_2 in second image
- 27 numbers, 26 dof

view_homog_matrix.exe

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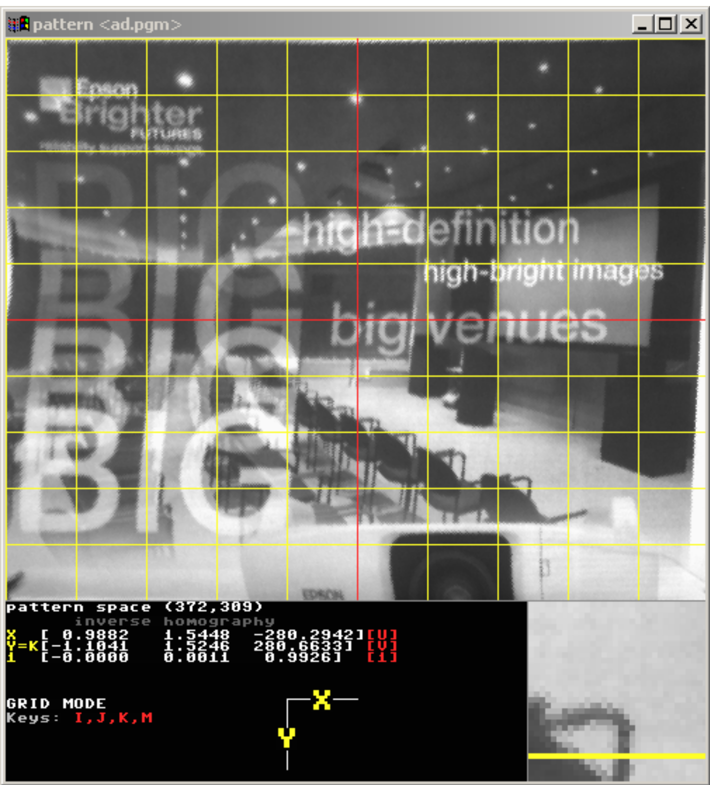
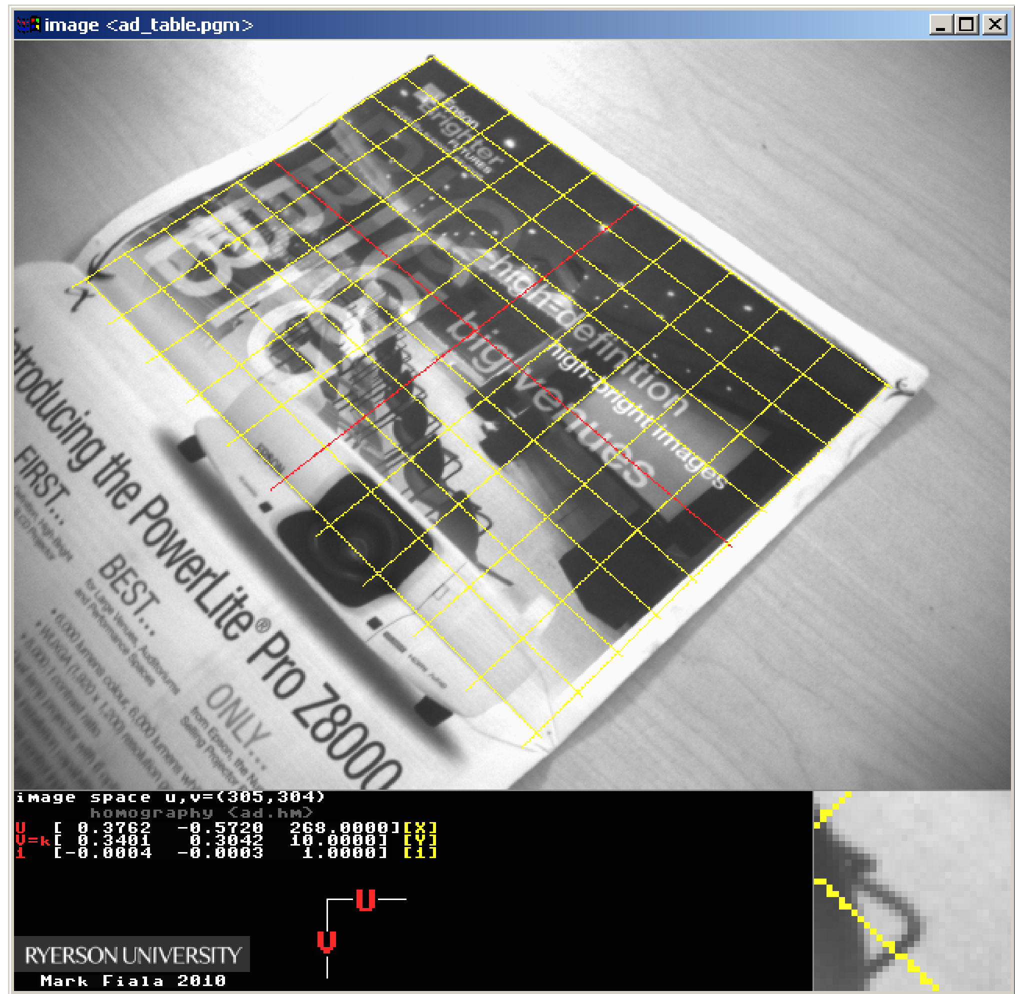
$$H = \begin{bmatrix} 0.3762 & -0.5720 & 268.0000 \\ 0.3401 & 0.3042 & 10.0000 \\ -0.0004 & -0.0003 & 1.0000 \end{bmatrix}$$





Recover Homography matrix from correspondences

$$\begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix}$$

$$\mathbf{u_2} = \frac{H_{11} u_1 + H_{12} v_1 + H_{13}}{H_{31} u_1 + H_{32} v_1 + H_{33}}$$

$$\mathbf{v_2} = \frac{H_{21} u_1 + H_{22} v_1 + H_{23}}{H_{31} u_1 + H_{32} v_1 + H_{33}}$$

Multiply up denominators

$$H_{31} \mathbf{u_1 u_2} + H_{32} \mathbf{v_1 u_2} + H_{33} \mathbf{u_2} = H_{11} \mathbf{u_1} + H_{12} \mathbf{v_1} + H_{13}$$

$$H_{31} \mathbf{u_1 v_2} + H_{32} \mathbf{v_1 v_2} + H_{33} \mathbf{v_2} = H_{21} \mathbf{u_1} + H_{22} \mathbf{v_1} + H_{23}$$

Solving for AX=0 -gives us 2 equations/correspondence –need 4 correspondences

$$H_{31} \mathbf{u_1 u_2} + H_{32} \mathbf{v_1 u_2} + H_{33} \mathbf{u_2} - H_{11} \mathbf{u_1} - H_{12} \mathbf{v_1} - H_{13} = 0$$

$$H_{31} \mathbf{u_1 v_2} + H_{32} \mathbf{v_1 v_2} + H_{33} \mathbf{v_2} - H_{21} \mathbf{u_1} - H_{22} \mathbf{v_1} - H_{23} = 0$$

Each correspondence provides two rows of A matrix

$$\begin{matrix} [-\mathbf{u_1} & -\mathbf{v_1} & -1 & 0 & 0 & 0 & \mathbf{u_1 u_2} & \mathbf{v_1 u_2} & \mathbf{u_2}\,] \\ [\;0 & 0 & 0 & -\mathbf{u_1} & -\mathbf{v_1} & -1 & \mathbf{u_1 v_2} & \mathbf{v_1 v_2} & \mathbf{v_2}\,] \end{matrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ \ldots \end{bmatrix}$$

Recover Homography matrix from correspondences

Each correspondence provides two rows of A matrix

$$\begin{matrix} [-u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1 u_2 & v_1 u_2 & u_2\,] \\ [\;0 & 0 & 0 & -u_1 & -v_1 & -1 & u_1 v_2 & v_1 v_2 & v_2\,] \end{matrix}$$

8 equations, 9 unknowns (from 4 correspondences)

$$\begin{bmatrix} -u_{11} & -v_{11} & -1 & 0 & 0 & 0 & u_{11}u_{21} & v_{11}u_{21} & u_{21} \\ 0 & 0 & 0 & -u_{11} & -v_{11} & -1 & u_{11}v_{21} & v_{11}v_{21} & v_{21} \\ -u_{12} & -v_{12} & -1 & 0 & 0 & 0 & u_{12}u_{22} & v_{12}u_{22} & u_{22} \\ 0 & 0 & 0 & -u_{12} & -v_{12} & -1 & u_{12}v_{22} & v_{12}v_{22} & v_{22} \\ -u_{13} & -v_{13} & -1 & 0 & 0 & 0 & u_{13}u_{23} & v_{13}u_{23} & u_{23} \\ 0 & 0 & 0 & -u_{13} & -v_{13} & -1 & u_{13}v_{23} & v_{13}v_{23} & v_{23} \\ -u_{14} & -v_{14} & -1 & 0 & 0 & 0 & u_{14}u_{24} & v_{14}u_{24} & u_{24} \\ 0 & 0 & 0 & -u_{14} & -v_{14} & -1 & u_{14}v_{24} & v_{14}v_{24} & v_{24} \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = \mathbf{0}$$

AX=b **X**=column vector of homography matrix elements

Get last vector using SVD

Image Rectification using a homography matrix

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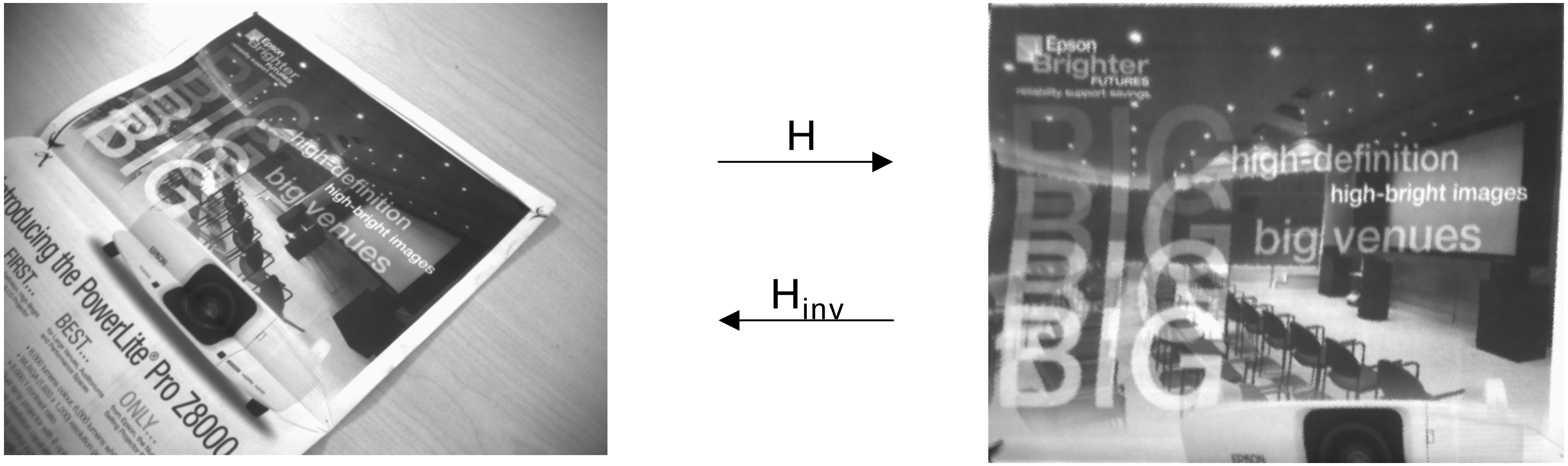


Correspondences

```
u11=268;  v11=10;   u21=0;   v21=0;
u12=558;  v12=220;  u22=499; v22=0;
u13=46;   v13=152;  u23=0;   v23=399;
u14=334;  v14=442;  u24=499; v24=399;
```


```
u11=268;  v11=10;   u21=0;   v21=0;
u12=558;  v12=220;  u22=499; v22=0;
u13=46;   v13=152;  u23=0;   v23=399;
u14=334;  v14=442;  u24=499; v24=399;
```


```
a1 =[-u11, -v11, -1, 0, 0, 0,  u11*u21, v11*u21, u21];
a2 =[0, 0, 0,  -u11, -v11, -1, u11*v21, v11*v21, v21];
a3 =[-u12, -v12, -1, 0, 0, 0,  u12*u22, v12*u22, u22];
a4 =[0, 0, 0,  -u12, -v12, -1, u12*v22, v12*v22, v22];
a5 =[-u13, -v13, -1, 0, 0, 0,  u13*u23, v13*u23, u23];
a6 =[0, 0, 0,  -u13, -v13, -1, u13*v23, v13*v23, v23];
a7 =[-u14, -v14, -1, 0, 0, 0,  u14*u24, v14*u24, u24];
a8 =[0, 0, 0,  -u14, -v14, -1, u14*v24, v14*v24, v24];
```

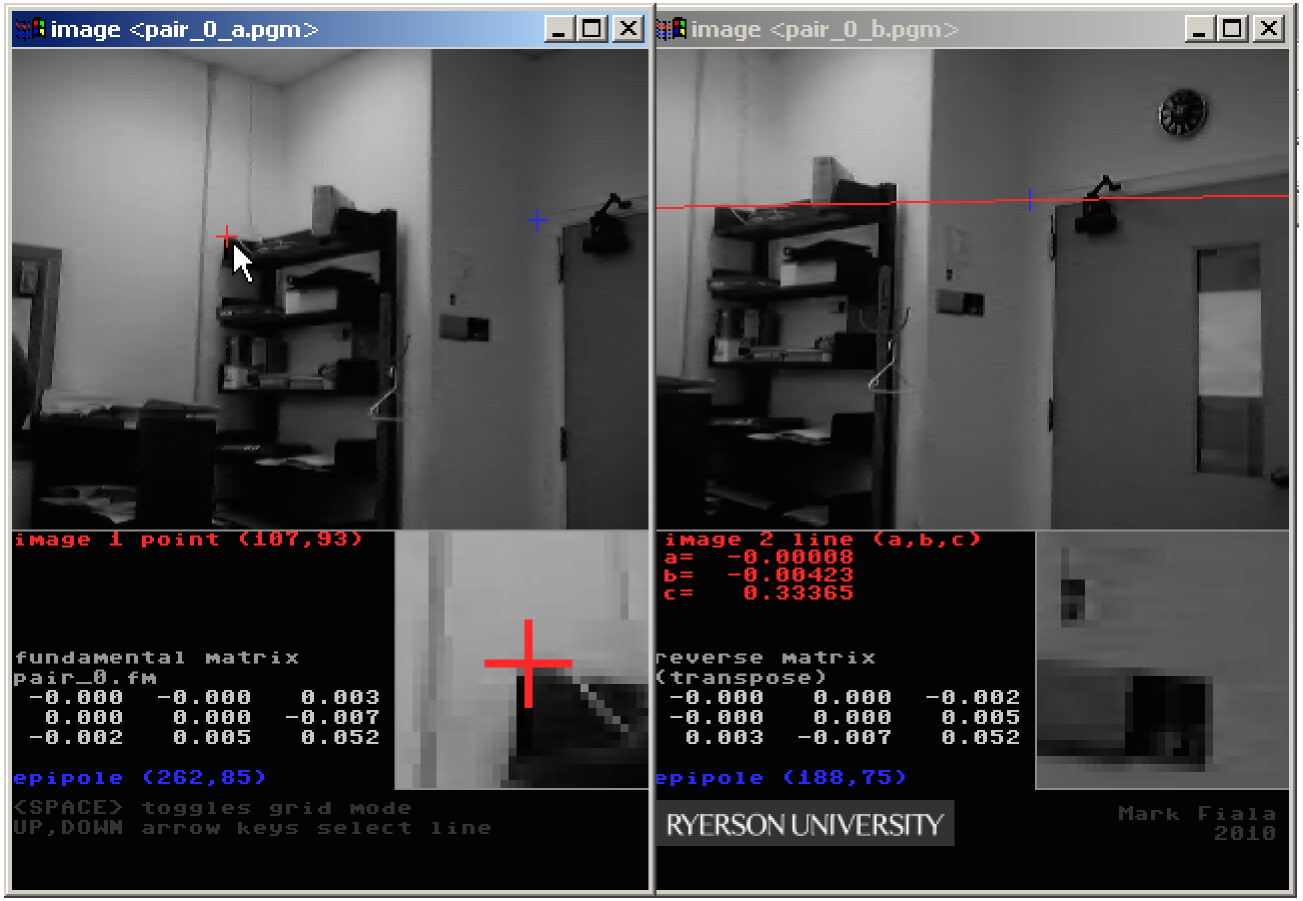
```
[u,d,v]=svd(a)

xtemp=v(8*9+1:9*9)'
x=xtemp/xtemp(9)
H=[x(1),x(2),x(3);x(4),x(5),x(6);x(7),x(8),x(9)]
```

```
H =

    0.9956    1.5566 -282.3961
   -1.1124    1.5362  282.7675
   -0.0000    0.0011    1.0000


Hinv =

    0.3762   -0.5720  268.0000
    0.3401    0.3042   10.0000
   -0.0004   -0.0003    1.0000
```

See matlab_lec9_solve_for_homog_matrix.txt

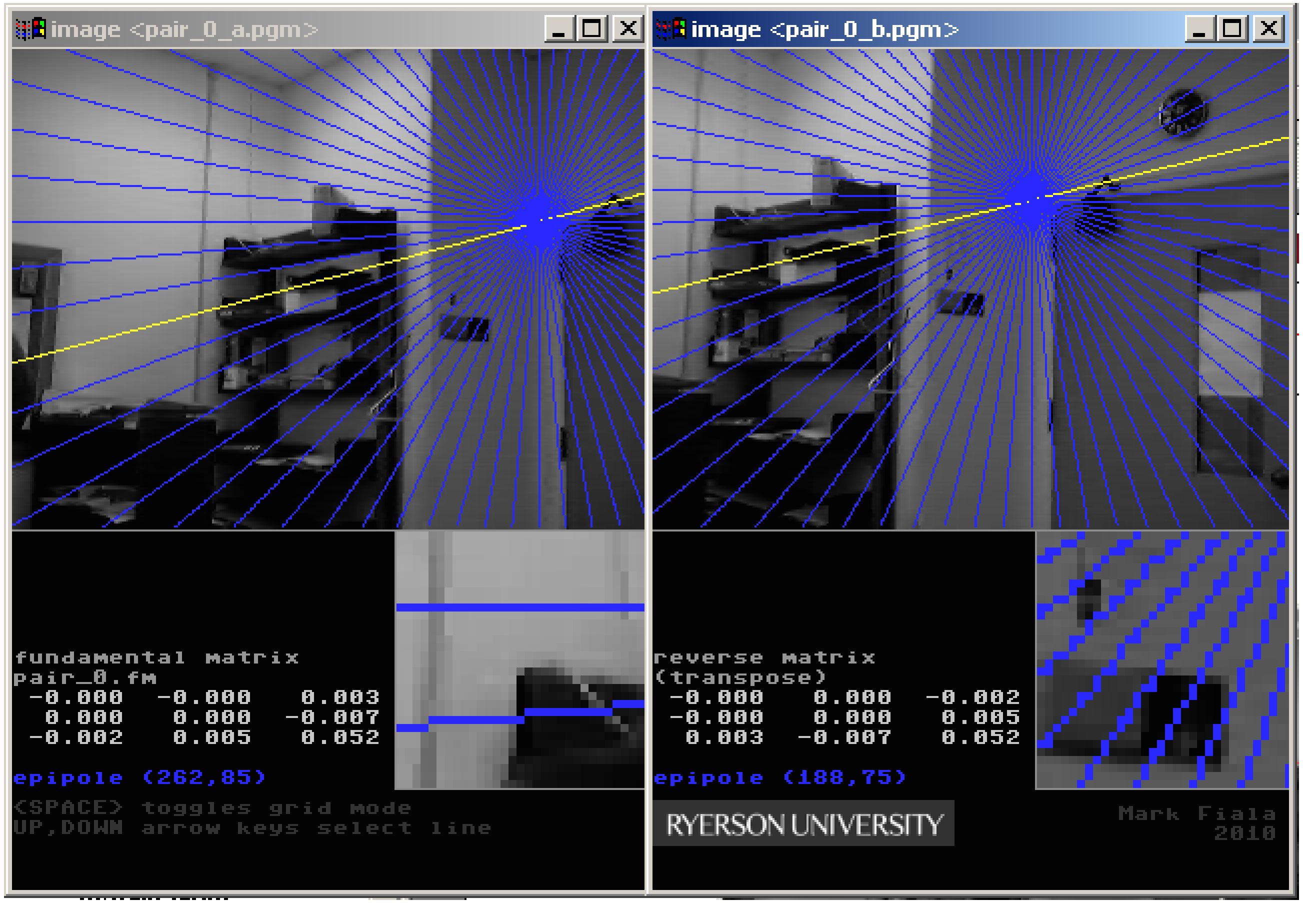
view_fund_matrix.exe

- view_homog_matrix.zip on course and Dr. Fiala webpage
- Point in left image (red cross, white arrow) maps to red line in right image
- Notice corner of shelves (red cross in left image) lies on line in right image
- Blue crosses are epipoles = projection of focal point of other camera
- Transpose of fundamental matrix is the reverse fundamental matrix mapping points in right image to lines in left (unlike reverse=matrix inverse with homographies)



view_fund_matrix.exe

- Space bar turns on grid mode – shows epipolar lines
- Epipolar lines radiate from epipoles



Fundamental matrix for rectified images

- To do 'proper' stereo disparity, we needed rectified images
- images where a point (x1,y) in the 1st image appears at (x2,y) in the 2nd image (y is the same)
- Fundamental matrix is:

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{matrix}$$

Recover Fundamental matrix from correspondences

$$[\,u_2 \;\; v_2 \;\; 1\,] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \mathbf{0}$$

Solving for AX=0 -gives us 1 equation/correspondence –need 8 correspondences

$$\begin{aligned} & F_{11}\mathbf{u_1 u_2} + F_{12}\mathbf{v_1 u_2} + F_{13}\mathbf{u_2} \\ & + F_{21}\mathbf{u_1 v_2} + F_{22}\mathbf{v_1 v_2} + F_{23}\mathbf{v_2} \\ & + F_{31}\mathbf{u_1} + F_{32}\mathbf{v_1} + F_{33} = 0 \end{aligned}$$

Each correspondence provides one rows of A matrix –need 8 correspondences

$$[\,\mathbf{u_1 u_2} \;\; \mathbf{v_1 u_2} \;\; \mathbf{u_2} \;\; \mathbf{u_1 v_2} \;\; \mathbf{v_1 v_2} \;\; \mathbf{v_2} \;\; \mathbf{u_1} \;\; \mathbf{v_1} \;\; \mathbf{1}\,] \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ \cdots \end{bmatrix}$$

Recover Fundamental matrix from correspondences

Each correspondence provides one rows of A matrix

$$[u_1u_2 \quad v_1u_2 \quad u_2 \quad u_1v_2 \quad v_1v_2 \quad v_2 \quad u_1 \quad v_1 \quad 1]$$

8 equations, 9 unknowns (from 8 correspondences)

$$\begin{bmatrix} u_{11}u_{21} & v_{11}u_{21} & u_{21} & u_{11}v_{21} & v_{11}v_{21} & v_{21} & u_{11} & v_{11} & 1 \\ u_{12}u_{22} & v_{12}u_{22} & u_{22} & u_{12}v_{22} & v_{12}v_{22} & v_{22} & u_{12} & v_{12} & 1 \\ u_{13}u_{23} & v_{13}u_{23} & u_{23} & u_{13}v_{23} & v_{13}v_{23} & v_{23} & u_{13} & v_{13} & 1 \\ u_{14}u_{24} & v_{14}u_{24} & u_{24} & u_{14}v_{24} & v_{14}v_{24} & v_{24} & u_{14} & v_{14} & 1 \\ u_{15}u_{25} & v_{15}u_{25} & u_{25} & u_{15}v_{25} & v_{15}v_{25} & v_{25} & u_{15} & v_{15} & 1 \\ u_{16}u_{26} & v_{16}u_{26} & u_{26} & u_{16}v_{26} & v_{16}v_{26} & v_{26} & u_{16} & v_{16} & 1 \\ u_{17}u_{27} & v_{17}u_{27} & u_{27} & u_{17}v_{27} & v_{17}v_{27} & v_{27} & u_{17} & v_{17} & 1 \\ u_{18}u_{28} & v_{18}u_{28} & u_{28} & u_{18}v_{28} & v_{18}v_{28} & v_{28} & u_{18} & v_{18} & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

AX=b **X**=column vector of fundamental matrix elements

Get last vector using SVD

Full system fundamental matrix example

from http://www.csse.uwa.edu.au/~pk/Research/MatlabFns/Robust/example/index.html

Steps:

1. Interest point detection
2. Correlation matching
3. RANSAC search for fundamental matrix
4. Inlier/outliers labeled

Input images

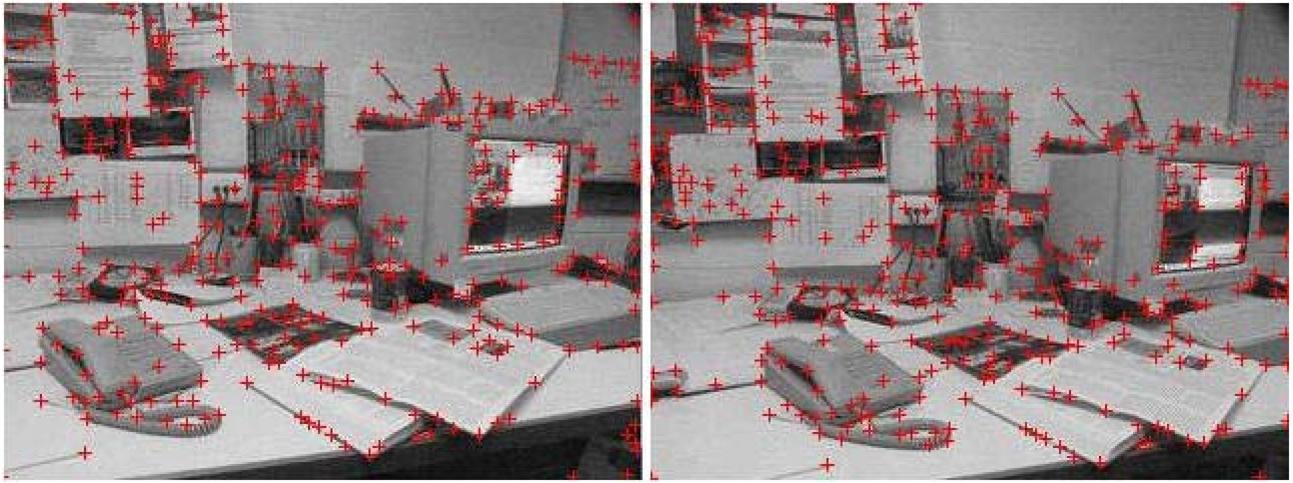
im1.jpg

im2.jpg

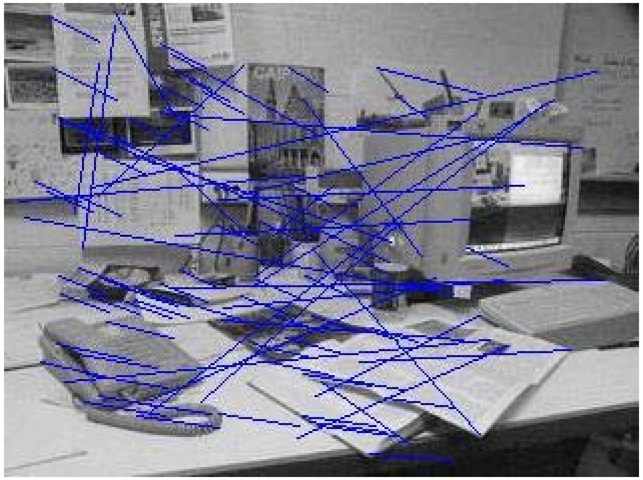
Full system fundamental matrix example

from http://www.csse.uwa.edu.au/~pk/Research/MatlabFns/Robust/example/index.html

Interest points found



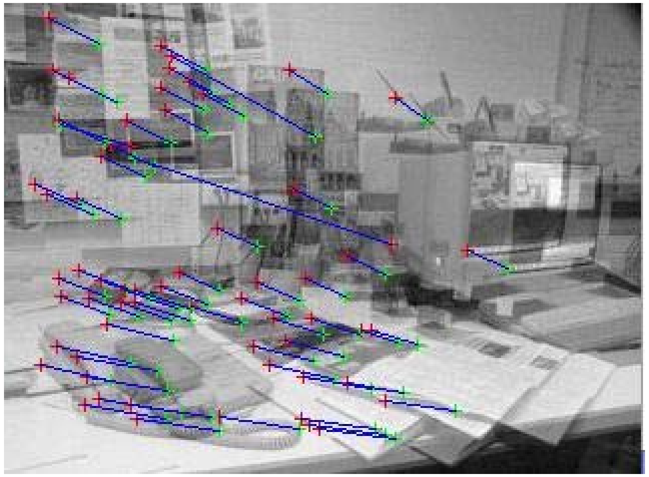
matches



Full system fundamental matrix example

from http://www.csse.uwa.edu.au/~pk/Research/MatlabFns/Robust/example/index.html

Inliers after Fund. Matrix found



Point to epipolar lines gui