Multi-Dimensional Scaling and Applications to Posture-Invariant Surface Processing

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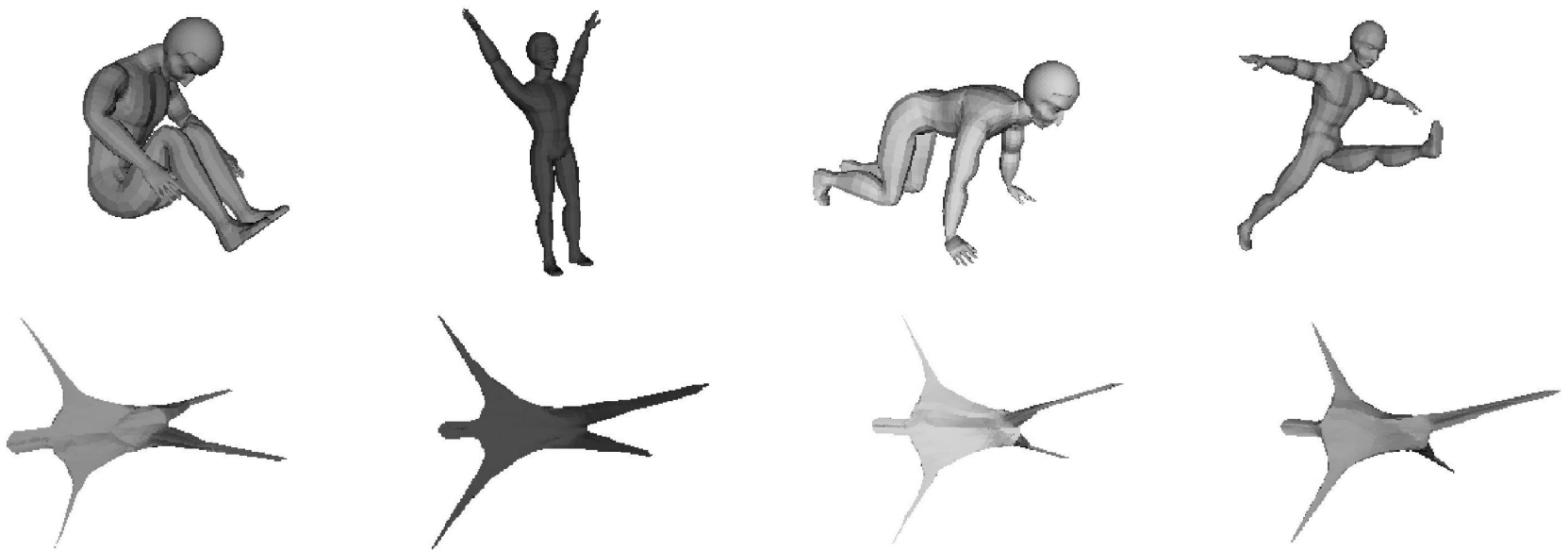


Outline

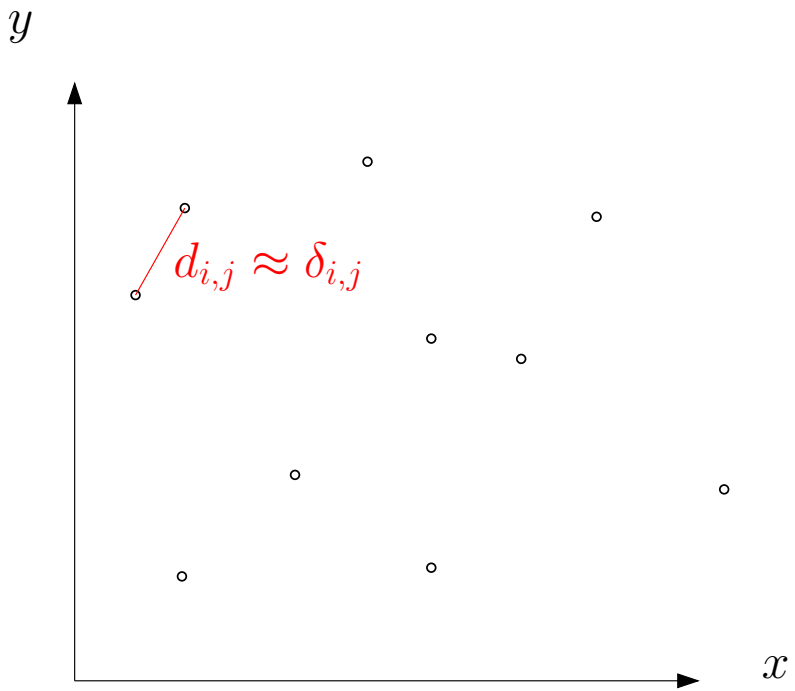
- Multi-Dimensional Scaling (MDS)
- Isometric Embeddings
- Applications of Isometric Embeddings
  - Surface Recognition
  - Face Recognition
  - Surface Correspondence
  - Feature Extraction
- Limitations of Isometric Embeddings

What is MDS?

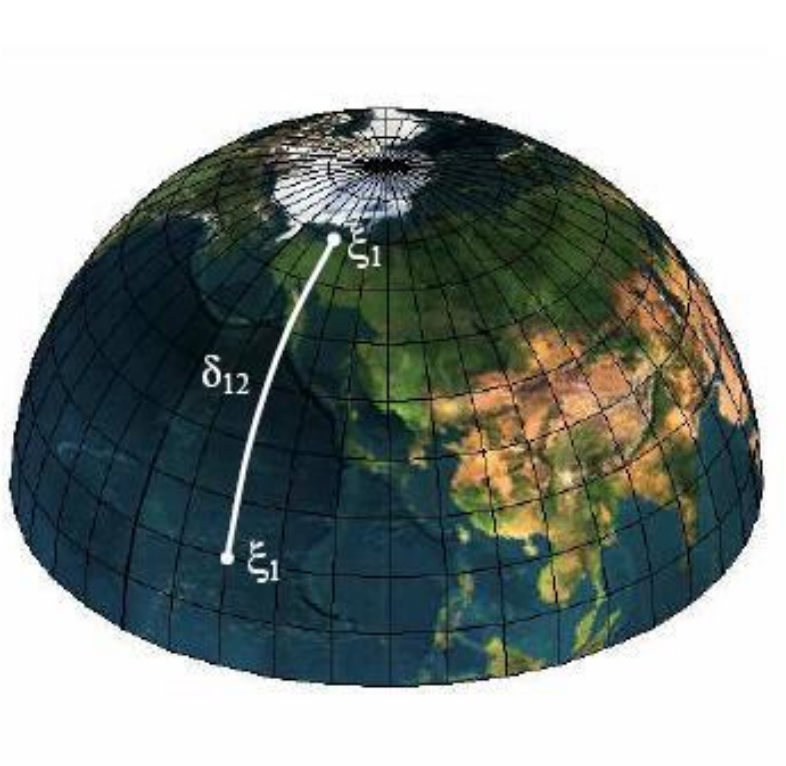
Given:
Dissimilarity matrix

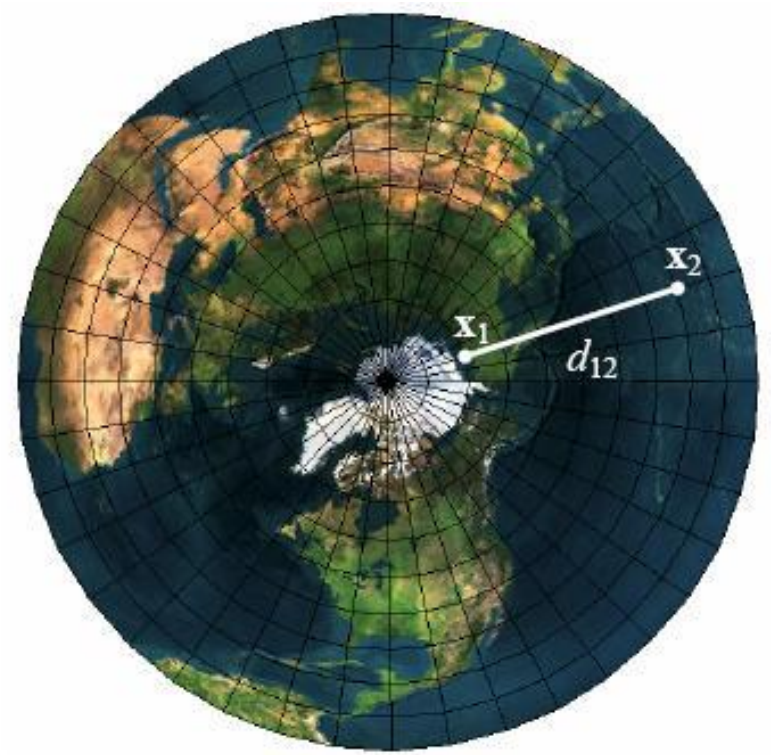
$$\Delta = \begin{bmatrix} \delta_{1,1} & \dots & \delta_{1,n} \\ \dots & \dots & \dots \\ \delta_{n,1} & \dots & \delta_{n,n} \end{bmatrix}$$

Find: $X = x_1, x_2, \dots, x_n \in \mathbb{R}^p$



Exact Solution Does Not Exist





$\Rightarrow$ Approximation required (Figure from [BBKY06])

Properties of Approximation

- Approximation known to be hard
- Objective function is nonlinear and non-convex
- Function, gradient, and Hessian require heavy computation
- Hessian is dense
- Solution is not unique (translation, rotation, reflections do not change value of stress function)

Methods for MDS

- Classical MDS [Gow66] (spectral method)
- Least-squares MDS [CC01] (gradient descent method)
- Fast MDS [FL95] (heuristic based on projecting points to hyperplanes)
- Generalized MDS [BBK06] (embedding space is an arbitrary surface)
- Many more methods, see Cox and Cox [CC01]

Extrinsic vs. Intrinsic Geometry

Extrinsic geometry

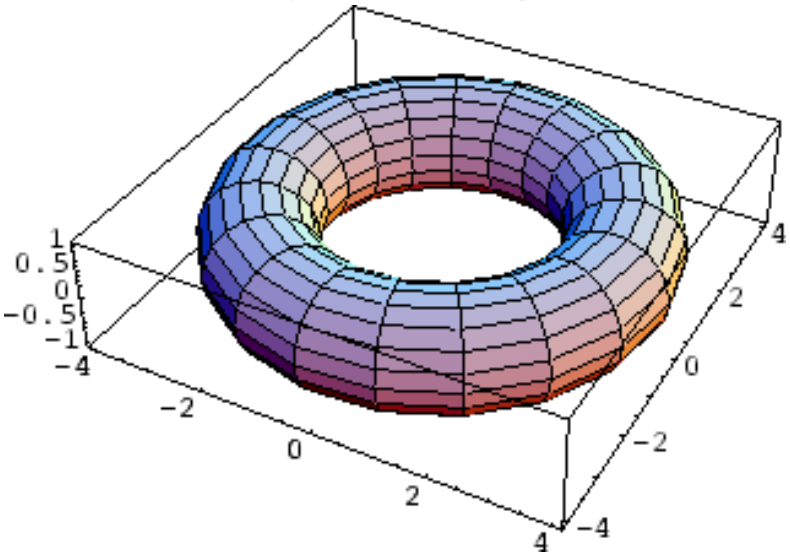


Figure created with Mathematica

Intrinsic geometry

Escher's Moebius strip

Isometry

Figure from [EK03]

S, Q : smooth Riemann manifolds
Mapping

$$\varphi : S \to Q$$

isometry if and only if geodesic distances are preserved.

Isometric Embedding

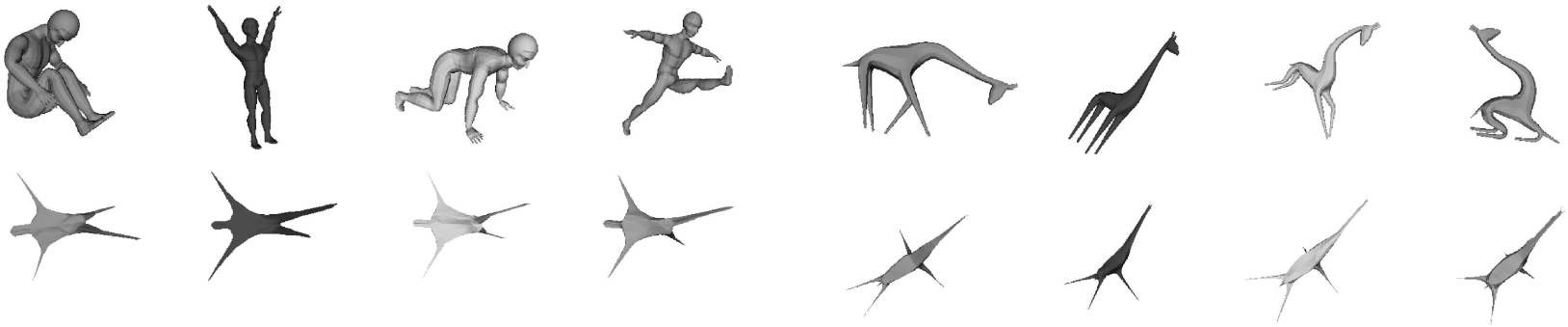
Isometric Embedding

Find a representation of the intrinsic geometry of a Riemannian manifold S in an embedding space Q with simple extrinsic geometry (often: $\mathbb{R}^3$).

How does MDS help?

We use geodesic distances as dissimilarities.

Surface Recognition [EK03]



Top: original surfaces. Bottom: canonical forms. Figures from [EK03]

Face Recognition [BBK05, BBK03]

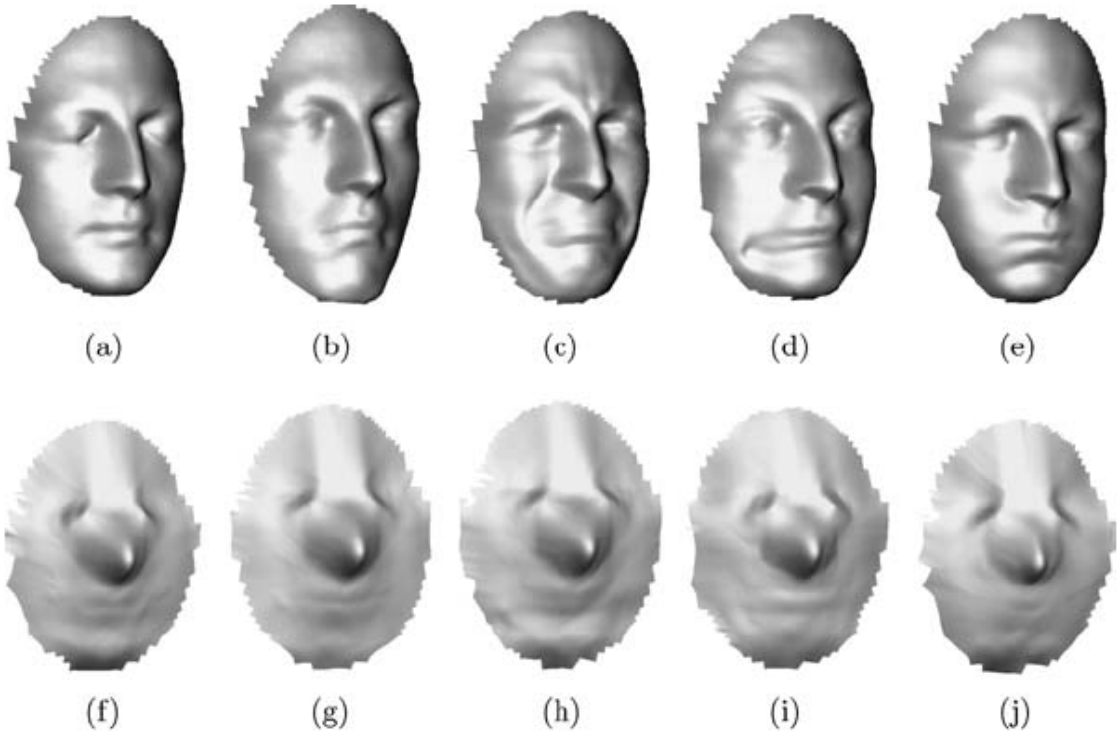
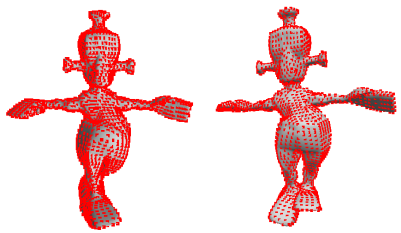


Figure from [BBK05]

Surface Correspondence [WSAB07]

- Given two incomplete manifold meshes $S^{(0)}$ and $S^{(1)}$



- Compute a canonical form in $\mathbb{R}^k$

- Compute correspondence in MDS space using rigid alignment

Surface Correspondence [BBK06]

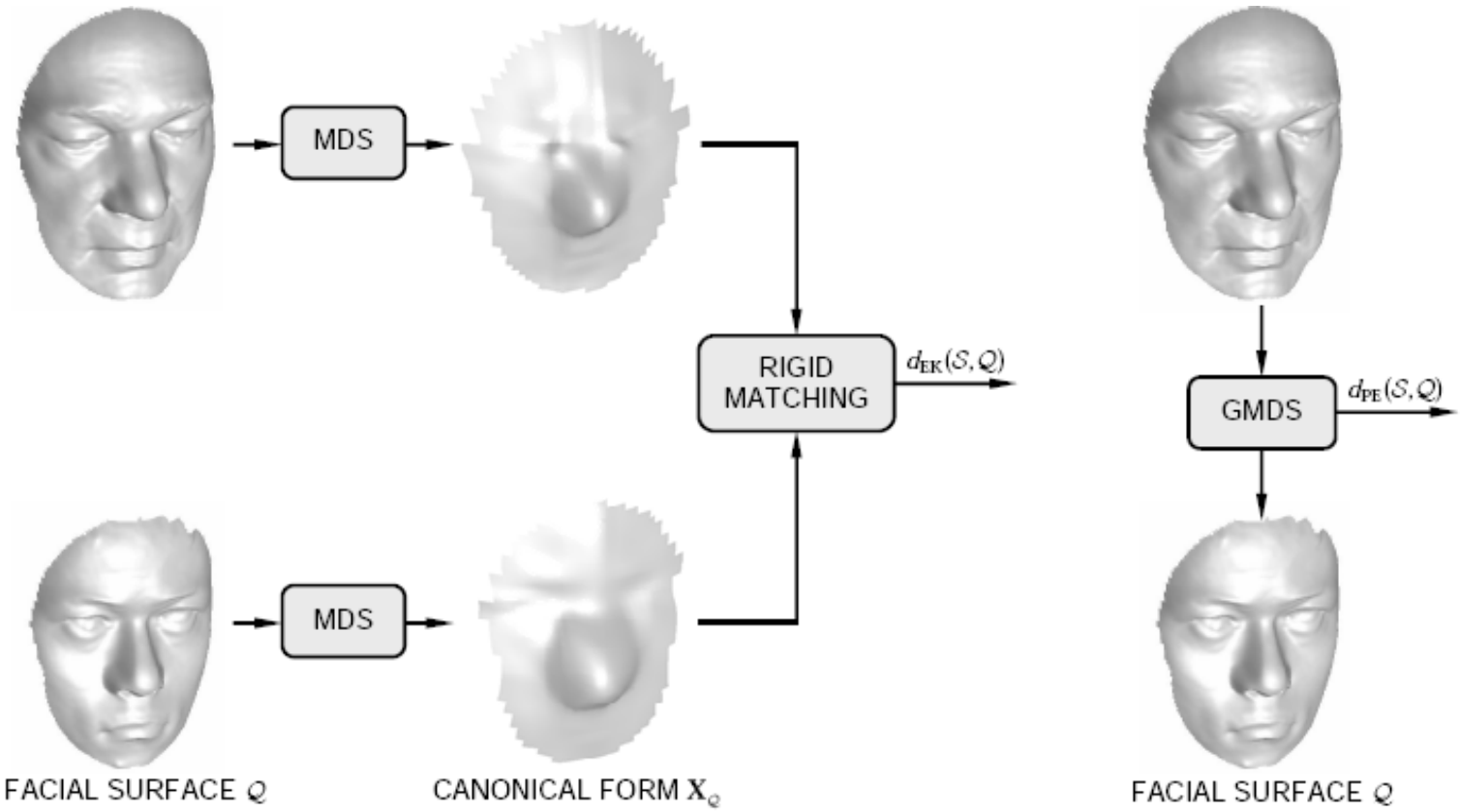
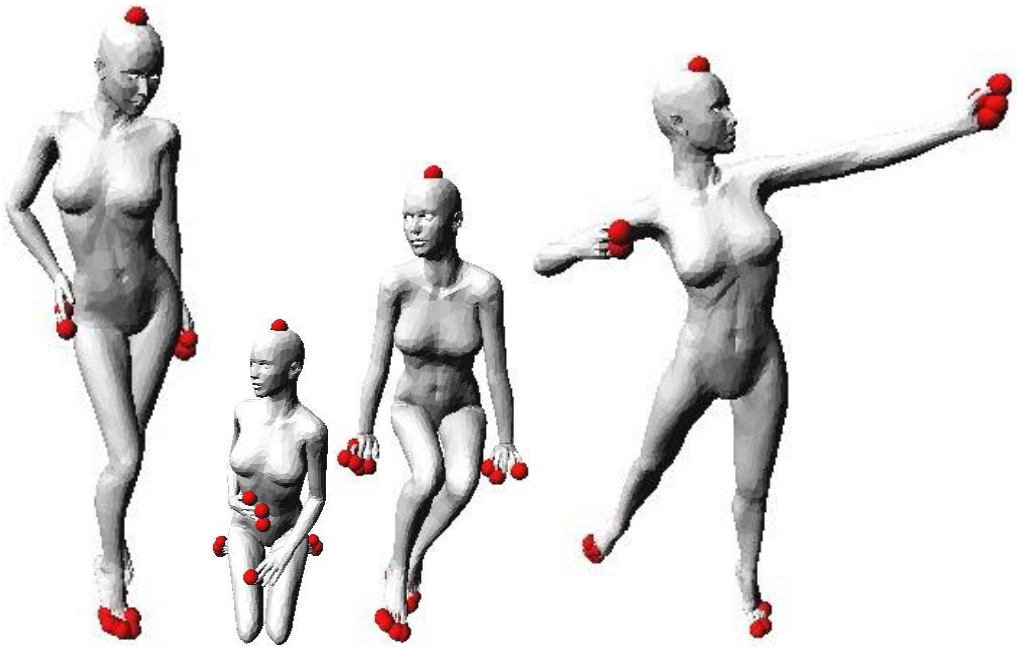


Figure from Bronstein et al. [BBK06].

Feature Extraction [WAS10]

Compute features as points with unusual Spin images [JH99] on canonical form.



Limitations

- Symmetric alignments
- Surfaces with non-Euclidean intrinsic geometries
- Large holes
- Outliers


धन्यवाद
Hindi
多謝
Traditional Chinese
ขอบคุณ
Thai
Спасибо
Russian
Gracias
Spanish
شكرا
Arabic
Thank You
Obrigado
Brazilian Portuguese
Grazie
Italian
Danke
German
Merci
French
நன்றி
Tamil
多谢
Simplified Chinese
감사합니다
Korean
ありがとうございました
Japanese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