CONSTRAINT-BASED MODELING OF THREE-DIMENSIONAL SHAPES

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ABSTRACT

This paper shows that constraint-based modeling, so far perceived primarily as a graphics technique for man-machine interaction, also provides a viable method for the modeling of complex surfaces. The idea of constraint-based modeling of three-dimensional shapes is described and illustrated by examples. Difficulties related to the practical application of this idea are discussed, and methods for overcoming them are outlined. A potential of the constraint-based approach to the modeling of shapes found in nature is indicated.

RESUME

Dans les systèmes infographiques à contraintes développés jusqu'à présent, les contraintes géométriques étaient utilisées surtout en qualité d'une technique d'interaction. Cependant, les mêmes contraintes peuvent former aussi une base pour modeler des objects à trois dimensions. Cet article présente l'idée principale du modelage à l'aide des contraintes et l'illustre avec des examples. L'application de la méthode pour modeler des surfaces complexes est mise en évidence. Les problèmes numeriques associés sont discutés, et une technique pour les attenuer par une decomposition hiérarchique du modèle est introduite. L'application potentielle de la méthode pour modeler des formes de nature est indiquée.

Keywords: constraint-based modeling, polygon meshes, free-form surfaces.

1. INTRODUCTION

> Of all the constraints of Nature, the most far-reaching are imposed by space.
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> Peter Stevens, *Patterns in nature.*

One common computer graphics technique for representing three-dimensional objects uses polygon meshes. A mesh is defined as a set of connected, polygonally bounded planar surfaces. Polyhedra are examples of meshes, but the notion of a mesh is more general. In particular, it also includes polygonal approximations of curved surfaces.

A mesh description consists of a specification of vertices, edges and faces. Known methods of mesh description require the positions of all vertices to be explicitly specified in a system of coordinates [Foley and van Dam 1983]. This is convenient in many situations, for instance, when a mesh is rendered. However, other parameters may be more convenient to use when a mesh is modeled. For example, consider descriptions of a regular tetrahedron. In terms of edges its definition is trivial - the tetrahedron must have four edges of equal length. In contrast, the description of a regular tetrahedron in terms of vertices is by far less intuitive, since their coordinates cannot be specified without arduous calculations.

Mesh definition by specifying the lengths of edges falls into the category of constraint-based modeling. Instead of specifying vertices directly, a set of constraints, or relations between vertices, is defined. The idea of specifying geometric figures using constraints is not new to computer graphics. It was first implemented in Sketchpad [Sutherland 1963], and followed in several other interactive graphics systems [Knuth 1979, Borning 1981, Van Wyk 1982, Nelson 1985]. Due to its intuitive character, constraint-based modeling was used there primarily as the basis for man-machine interaction. The possibility of building a constraint-based system for the purpose of computer aided design was indicated by Lin, Gossard and Light [1981].

This paper presents a new application of constraint-based modeling - definition of complex three-dimensional shapes.

2. UNSTRUCTURED MODELING

Various types of constraints can be used when describing a mesh. For example, they may characterize vertices as co-linear or co-planar, specify areas of faces, fix the angles between edges and faces, etc. The mesh representation described in this paper uses distances between points as the main form of constraint. Additionally, lines can be specified as parallel to any plane of the system of coordinates (*xy*, *xz* or *yz*), and selected coordinates of vertices can be explicitly given. Explicit specification of some coordinates and directions is necessary to position a rigid object in space, so that it cannot translate nor rotate. Thus, the complete description of a mesh containing n vertices consists of: