GRAPHICAL APPLICATIONS OF L-SYSTEMS

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ABSTRACT.

A new method for generating pictures is presented and illustrated with examples. The idea is to generate a string of symbols using an L-system, and to interpret this string as a sequence of commands which control a "turtle". Suitable generalizations of the notions of the L-system and of a turtle are introduced. The resulting mathematical model can be used to create a variety of (finite approximations of) fractal curves, ranging from Koch curves, to classic space-filling curves, to relatively realistic-looking pictures of plants and trees. All these pictures are defined in a uniform and compact way.

RESUME.

Une nouvelle méthode pour engendrer des images est presentée et illustrée avec des exemples. Cette méthode comprend deux étapes. On commence par engendrer une séquence de symboles avec un L-système. Ensuite on utilise cette séquence pour contrôler les mouvements d'une tortue qui trace l'image en question. Les notions d'un L-système et d'une tortue sont généralisées pour mieux correspondre au but de la création des images. Le modèle mathématique résultant s'applique à la création d'une large variété d'objets fractals, y compris des courbes de Koch, des courbes qui remplissent tout une aire plaine, ainsi que des images relativement réalistes des plantes et des arbres. Toutes ces images sont définies d'une manière homogène et compacte.

KEYWORDS: L-systems, turtle geometry, fractals, space-filling curves, plants, trees.

1. INTRODUCTION.

Rewriting systems can be used to generate pictures in two different ways. In the first case, rewriting systems operate directly on two-dimensional objects, such as arrays [Kirsch 1964, Dacey 1970], graphs [Rosenfeld and Milgram 1972, Pfaltz 1972], or "shapes" [Gips 1975, Stiny 1975]. In the second case, string grammars (in the broad sense of the word, including parallel rewriting systems) are used to define strings of symbols. A graphic interpretation function subsequently maps these strings into pictures. This paper describes a method for generating pictures based on this second approach. After the idea of applying string grammars to pictures is put into a historic perspective in Section 2, attention is focused on L-systems [Lindenmayer 1968]. The necessary definitions related to 0L-systems are collected in Section 3. Section 4 adapts the notion of a "turtle" [Papert 1980, Abelson and diSessa 1982] to the purpose of graphical interpretation of strings, and presents examples of pictures generated by 0L-systems under this interpretation. Section 5 extends this basic approach in two directions: by generalizing the notion of the L-system, and by increasing the range of string symbols interpreted by the turtle. Section 6 presents conclusions and lists several open problems.

2. THE HISTORICAL BACKGROUND.

The idea of describing pictures using formal (string) languages emerged a few years after Chomsky established the fundamental concept of a phrase-structure grammar. Narasimhan [1962, 1966] and Ledley [1964, 1965] are credited with the first results in this area. Their interest was in the recognition of handwritten characters and chromosomes, respectively. An approach designed for describing a wider class of pictures using string grammars was proposed by Shaw [1969]. For a survey of these early results, see Fu [1980].

The early research concentrated on picture recognition. Pictures were described as strings of symbols which represented selected primitives, such as straight segments, sharp V-turns, wide U-turns or branches. In some cases, relations between picture elements, such as ABOVE, BELOW, of INSIDE, were also considered as primitives. The actual picture recognition was performed by parsing the resulting strings.

In the case of picture generation, the correspondence between string symbols and picture primitives must be specified in more detail. The first such specification, known as chain coding, was developed by Freeman [1961]. Feder [1968] showed that the languages of chain codes describing such classes of figures as straight lines of arbitrary slope, circles of arbitrary radius, or convex figures in a plane, are all context sensitive. It was subsequently pointed out (for example, by Fu [1980]) that even intuitively simpler classes of pictures, for example the set of all rectilinear squares in an integer grid, correspond to context-sensitive chain-code languages. To a certain degree, this discouraged a further study of chain-code languages, for the context-sensitive grammars are usually difficult to construct and do not provide an intuitively clear description of languages. Neverthe-