A COMPUTATIONAL THEORY OF 3D SHAPE RECONSTRUCTION FROM IMAGE CONTOURS

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ABSTRACT

The computational theory of 3D shape reconstruction from image contours proposed in this paper is based on the variational principles and has the theoretical framework suggested recently by Poggio [8]. By this theory, 3D shapes can be reconstructed from image contours by general physical constraint assumptions, namely the assumptions of minimum potential energy, isotropism and homogeneity of the material, and properly defined energy functionals. It is assumed that contours have been classified as surface discontinuity boundary contours, surface contours and extremal boundaries. Minimization of the energy functionals tends to maximize the symmetry and orthogonalize the surface junctions of the reconstructed object. Some early findings are obtained as the natural results of the theory. Theoretical developments and experimental results are successful and promising.

EXTENDED SUMMARY OF THE THEORY

One important function of early vision is the reconstruction of a 3D representation of a scene from 2D images. Stereopsis and structure from motion are the most explored in vision studies. Stereopsis and structure from motion require multiple images, but humans have the ability to perceive (with illusion) the 3D environment with only one eye or from a single picture.

There exist many sources of information about surfaces in an image such as texture, shading, shadow, etc. [9,12,13,17], but those methods are only applicable for certain special situations. It has been shown that shape reconstruction from contours is significantly more powerful than shape reconstruction from textures [7]. Barrow and Tenenbaum [11] argued that shape reconstruction from boundary contours is of fundamental importance in explaining surface perception and more important than shape reconstruction from shading. Steven [14] showed that surface contours also play an important role in shape reconstruction from image.

Theoretical studies [8] showed that the computational, ill-posed nature of early vision problems leads naturally to the application of the mathematical theory of regularizing ill-posed problems for solving them in terms of variational principles that enforce general physical constraints derived from a physical analysis of the problem. The constraints should be derived as a natural consequence of the physical laws governing the world we live in. The results of this research will not only impact our understanding of the early visual system in biological organisms, but also lead to development of computational algorithms and hardware designs for machine vision.

The fact that the human visual system has definite and consistent interpretations of contour images shows that it exploits some implicit assumption about the world. Without any knowledge of the nature of the process which generated the 3D shape, it is reasonable to assume that the given 2D contour is most likely to correspond to the projectively equivalent 3D shape with minimum potential energy. It is well known from classical mechanics that a physical system is stable if and only if its total potential energy is minimal. In many cases, it is also justified on the grounds that the surfaces tend to assume smooth and minimal energy configurations. Because there is no information available in the image contours about the material of the surface, the only reasonable assumption is that the surface material is isotropic and homogeneous. The computational theory of shape reconstruction from contours proposed in this paper is based on the variational principles in terms of general physical constraint assumptions : (1). the minimum energy principle; (2). isotropism and homogeneity of the material, i.e., the uniformity of the energy distribution.

To deform a system with minimum total potential energy to a non-minimum energy system, external energy has to be converted into potential energy in the system. As is well known, the differential equations describing a system in non-minimum energy state are far more complicated than the equations describing the system in its minimum energy state. Non-uniformity of energy distribution represents information about the system. Thus we can draw a correspondence between energy, energy distribution and information. Therefore the interpretation of the image contours by this theory is a minimum information interpretation in some sense.

Earlier studies such as [3,4,5,6,9,10,11,12,13,14,17] motivate and support the theory proposed in this paper. Barrow and Tenenbaum [11] optimized a smoothness measure to reconstruct planar curves and polyhedra. The optimization criterion used in [11] for continuous curves and straight lines are different, whereas a complete theory of shape