Algebraic Error Analysis for Surface Curvatures of 3-D Range Images

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ABSTRACT

The method of calculating the Gaussian and mean curvatures of 3-D range images using differential geometry and approximation theory is given. An algebraic error analysis for the calculated curvatures is then presented. Upper bounds for the curvature error terms are obtained as a function of the window size used in the curvature calculation and of the noise standard deviation. The error analysis results are used to illustrate the effect of noise on the segmentation results using curvature sign labels. A segmentation procedure for 3-D range images using a 3-sign label scheme is proposed. Experimental results are presented.

1. INTRODUCTION

Surface curvatures of 3-D range images are invariant under translation and space rotation. They are powerful tools in image description and segmentation.

Normally the range data is contaminated with noise and unfortunately, the calculated surface curvatures for 3-D range images are very sensitive to noise. As a result, when the surface is segmented using the curvature sign labels, spurious patches are created in the segmentation.

In this paper, the method of calculating the surface Gaussian and mean curvatures of 3-D range images using differential geometry and approximation theory is given. The sign label scheme used for segmenting the 3-D range images is explained. An algebraic error analysis for the calculated curvatures is then presented. Upper bounds for the curvature error terms are obtained as a function of the window size used in the curvature calculation and of the noise standard deviation.

The error analysis results are used to illustrate the effect of noise on the segmentation results using curvature sign labels. A practical case shows that the boundaries of the spurious segmentation patches occur where the relative error terms in the calculated curvatures are high. A segmentation procedure for 3-D range images using a 3-sign label scheme is proposed. Experimental results are presented.

2. Gaussian and mean curvatures

At each point on a 3-D surface, there exist a direction of maximum normal curvature denoted by k_1 and an orthogonal direction of minimum normal curvature denoted by k_2. The Gaussian curvature K is defined as the product k_1k_2, while the mean curvature H is the mean $(k_1+k_2)/2$;

$$K = k_1k_2 \quad \text{and} \quad H = (k_1 + k_2)/2 \qquad (1)$$

Let the range image depth z be defined in terms of the surface coordinates u and v; $z = z(u,v)$. Let also z_u, z_v, z_{uu}, z_{uv} and z_{vv} be the partial derivatives of z with respect to u and v. The curvatures K and H are given by the following formulas [1,2].

$$K = \frac{z_{uu}z_{vv} - z_{uv}^2}{(1 + z_u^2 + z_v^2)^2} \qquad (2a)$$

and

$$H = \frac{z_{uu} + z_{vv} + z_{uu}z_v^2 + z_{vv}z_u^2 - 2z_uz_vz_{uv}}{2(1 + z_u^2 + z_v^2)^{1.5}} \qquad (2b)$$

In practice, the range image depth z is given in the form of discrete data points at integer (x,y) coordinates. It is common to fit a second degree surface in a least squares sense, to a window centered at each point of the range surface. The partial derivatives of the fitting surface at the centre of the window are taken as the partial derivatives of the range data at that point. The validity of this assumption is not questioned in this work and the results of this paper are based on this assumption.

Figure 1 shows the relative coordinates of the surface points (pixels) inside the 5x5 window and figure 2 shows the pixel labels inside the 5x5 window.

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|  (-1,-2) (-1,-1) (-1,0) (-1,1) (-1,2)
i   (0,-2)  (0,-1)  (0,0)  (0,1)  (0,2)
    (1,-2)  (1,-1)  (1,0)  (1,1)  (1,2)
    (2,-2)  (2,-1)  (2,0)  (2,1)  (2,2)
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Figure 1. Relative coordinates of the pixels inside a 5x5 window.