Curvature Morphology

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Abstract

The notion of curvature of planar curves has emerged as one of the most powerful for the representation and interpretation of objects in an image. Although curvature extraction from a digitized object contour would seem to be a rather simple task, few methods exist that are at the same time easy to implement, fast, and reliable in the presence of noise. In this paper we first briefly present a scheme for obtaining the discrete curvature function of planar contours based on the chain-code representation of a boundary. Secondly, we propose a method for extracting important features from the curvature function such as extrema or peaks, and segments of constant curvature. We use mathematical morphological operations on functions to achieve this. Finally, on the basis of these morphological operations, we suggest a new scale-space representation for curvature named the *Morphological Curvature Scale-Space*. Advantages over the usual scale-space approaches are shown.

Sommaire

Le concept de courbure appliqué aux courbes et contours planaires est reconnu comme un outil des plus utiles pour la représentation et l'interprétation de projections d'objets dans une image numérique. L'obtention de la fonction de courbure à partir d'un contour d'objet peut sembler à première vue simple. Pourtant, très peu de méthodes existent qui soient à la fois aisément programmable, rapide en temps de calculs et robuste en présence de bruit ou de perturbations. Dans le présent article, nous présentons, dans un premier temps, une telle méthode, afin d'obtenir une fonction de courbure discrète sous une forme adéquate, ce sur la base de la représentation dîte du code de chaîne ("chain-code"). Par la suite, nous proposons une méthode pour extraire et identifier les caractéristiques morphologiques de la fonction de courbure, telles que les extrema et segments à courbure constante ou plateaux. Pour ce faire nous utilisons des opérateurs définis dans le cadre de la Morphologie Mathématique. Finalement, sur la base de ces opérateurs morphologiques, nous introduisons une nouvelle représentation multi-échelles de la fonction discrète de courbure. Les avantages de cette représentation sont démontrés.

Index terms: Shape Representation, Curvature Representation and Analysis, Morphological Operators, Multiscale Description, Multi-Dimensional Scale-Space.

1. Introduction

Shape representation and analysis is fundamental to computer vision and our interest in it is an outgrowth of a study of the dynamic changes in cell shape [15, 17]. Of the many approaches to shape that have been proposed, the notion of curvature of planar curves has emerged as one of the most powerful for the representation and interpretation of objects in an image [14]. Curvature is a measure of the rate of change in orientation at each point along a curve. There is psychophysical, physiological, as well as computational and mathematical support in favor of using curvature as a representation for contours. Curvature extrema seem to be used by the human visual system to segment contours into meaningful parts [2, 4, 13]. For example, endstopped neurons in the visual cortex can be interpreted as performing local curvature measurements [9]. Local estimations of curvature and tangent information are sufficient for the recovery of the trace of a curve in an image [24]. From differential geometry, the fundamental theorem of the local theory of curves states that any regular[1] planar curve is uniquely defined by its curvature [6]. Non-regular points of a curve are singular points where the curvature goes to infinity, that is, where the change in orientation is undefined. They correspond to visually salient points such as corners. Therefore, given the set of singular points of a contour, which can be represented as extrema of curvature, as well as the curvature values for all intervening points, a contour is uniquely defined. This representation by curvature is invariant to rigid motion, that is, with respect to translation or rotation.

In a typical computer vision system, discrete contours of objects are first extracted from an image. Curvature of these discretized contours is then approximated and used to detect important features of the boundary of an object. Although curvature extraction from an object contour would seem to be a rather simple task, few methods exist that are simultaneously easy to implement, fast and reliable in the presence of noise. In this communication we first briefly propose a scheme for obtaining the discrete curvature function of planar contours (opened or closed) based on the chain-code representation of a boundary [11]. This approach has been previously reported and we mention only our own attempts to optimize its implementation.

Secondly, we present a way of extracting important features from the curvature function. We are interested in localizing features such as extrema or peaks of curvature, points of inflection (i.e. zero-crossings of curvature), and segments of constant curvature that correspond to straight line segments or circular arcs of the boundary. Furthermore, we would like to be able to differentiate these features by their relative significance, that is, by the degree of their isolation from nearby features, their relative amplitude, and their scale or size. Consequently, we seek methods for segmenting the curvature function into its basic and significant events. To achieve this

[1] Regularity implies continuity of a curve and its derivatives.