# Computing 2D Motion of Boundaries from Correspondences of Points of Significant Curvature

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## Abstract

*Finding the parameters of 2-D motion of curves is posed as a correspondence problem between points of significant curvature along these curves. The algorithm developed for this purpose is simple and intuitive. It uses natural ordering of such points along curves, the consistency of the expansion factor, and the angle of rotation to constrain the search for the true pairings. The parameters of the motion – namely the expansion, translation and rotation - are then evaluated from the positions of matched points of significant curvature rather than from curvature values as done by other techniques. This novel technique is robust and copes particularly well with occlusion problems at curve ends. In particular, our technique is shown to be more accurate in determining the parameters than the Hough Transform.*

**Keywords:** 2D Motion, Correspondence, high curvature points, Affine Transform, Boundaries, Curves.

## 1 Introduction

An important task in computer vision and image processing is to determine the transformation which maps a *2-D* curve into another. Such transformations are of crucial importance to *2-D* recognition [4, 13, 2], motion and tracking. To this end, points of significant curvature have received special attention in the computation of the parameters of the transformation due to their local nature. A comprehensive comparison of curvature estimation methods has been reported in [15]. More recently, the results of a series of experiments on three different estimators of point curvature in varying degrees of noise were published [6].

Many methods have been proposed to evaluate the parameters of the transformation of a *2-D* curve into another. Such techniques can be classified into local and global: the latter are based on matching a global vector of features, and thus suffer from occlusion [12]; the former use local features (such as curvature), which depend only upon portions of objects [1, 7].

Cohen *et al.*, proposed a method for determining transformations between two curves. This method is based on the minimisation of energy which tends to preserve the matching of points of significant curvature, while ensuring a smooth field of displacement vectors everywhere [3]. However, this technique is suited for deformable curves, and is very expensive since it requires the solution of a second order partial differential equation. Furthermore, the method also requires a first guess to start the iterative solution of the differential equation, and does not explicitly solve for the parameters of the transformation.

Ma and Chen's technique is based on a Generalised Hough transform [8], and can be summarised as follows:

For a given pair of boundaries $M$ and its transform $N$, let $(m, n) \in M \times N$ be a pair of corresponding points on the continuous curves $M$ and $N$. Asserting that $m = (x(t), y(t))^T$ corresponds to $n = (u(t), v(t))^T$ gives:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = kR \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (1)$$

where $k$ is the expansion factor, $X_0 = (x_0, y_0)^T$ is the translation vector and $R$ is the rotation matrix:

$$R = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

The slope and curvature at a point $m = (x(t), y(t))^T$ on a continuous curve parametrised by $t$ are given by:

$$tg(\alpha) = \frac{y'}{x'} \quad (2)$$