Determining Optical Flow for Large Motions Using Parametric Models in a Hierarchical Framework

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Abstract

The problem of computing optical flow (image velocity) for large image motions has been addressed by a number of researchers, including Bergen et al. [5], who proposed using a number of parametric models in a hierarchical (pyramid) scheme to compute large motions using image intensity derivatives computed from "warped" image patches at each pyramid level. We have implemented a modified version of their algorithm for four parametric models (0^{th} order, i.e. assuming constant image velocity in a local neighbourhood, 1^{st} order, i.e. assuming at most an affine transformation of image velocity in a local neighbourhood, and 2^{nd} order, i.e. assuming image velocities are computed on either planar and curved environmental surfaces). While the description of Bergen et al.'s algorithm's is less detailed than desirable in some places, we followed it as faithfully as we could (with only minor modifications). We present two novel ideas in this paper. First, we present a quantitative analysis of the error obtained at each level in the pyramid (using Fleet's angle error measure [7]) and second, we propose four ways of thresholding on the optical flow field at each level in the pyramid, to avoid projecting bad image velocities down the pyramid. We investigate the use of condition numbers, determinants and eigenvalues of the least squares integration matrix or of Gaussian curvature of the local (warped) image patches as thresholds. We show that as velocity is propagated down the pyramid using our thresholding scheme it becomes more accurate for controlled optical flow field densities.

1 Introduction

A fundamental problem in Computer Vision is the computation of image motion, which is the perspective projection of individual 3D scene points moving relative to an observer (camera) onto a 2D planar imaging surface. Optical flow is an approximation of this 2D image motion, where a number of assumptions, such as that all illumination changes are due entirely to motion, the scene surfaces are Lambertian and image motion is locally translational, are made.

Another approach to computing image motion involves computing correspondences between "interest" points (candidate match points), such as corner points or other locally discriminable points in (usually) two images and using these correspondences as the flow. Generally, a smoothness constraint is imposed, for example, see the algorithms of Anandan [1] or Singh [17, 18] to ensure that a dense (100%) flow is obtained. In general, good interest points are difficult to find and without a smoothness constraint, sparse flows result, for example, see Barnard and Thompson [2]. A recent study [4] has shown that at least for slow motions (2-4 pixels/frames) local differential methods are better that correspondence methods, even those with smoothness constraints. As well, differential methods employing smoothness constraints, for example Horn and Schunck [11], weren't as good as local differential methods, for example, Lucas and Kanade [13], for slow image motions.

The approach to optical flow that we investigate here (reported in detail in a MSc thesis [12]) is local and hierarchical (allowing large image motions to be handled) and follows closely the work of Bergen et al. [5]. In contrast, there is the hierarchical differential work of Glazer [10, 9], which imposes a smoothness constraint at each level in the pyramid and is effectively hierarchical Horn and Schunck.

2 Background

The motion constraint equation is given as

$$I_x v_x + I_y v_y + I_t = 0, \tag{1}$$

where I_x, I_y and I_t are spatio-temporal intensity derivatives and $\mathbf{v} = (v_x, v_y)$ is the image velocity. For this equation to be applicable the motion must be locally translational, the intensity must be preserved over time and the image must be continuous over space and time in order to allow reliable derivatives to be computed (without undue aliasing effects). For large motions this last constraint is usually not satisfied.

The motion constraint equation defines a line in (v_x, v_y) space and any point on that line satisfies equation (1). The velocity with the smallest magnitude is normal to this line and is called the normal velocity,

$$\mathbf{v_n} = \frac{-I_t \nabla I}{||\nabla I||_2^2} = v_n \hat{\mathbf{n}}, \tag{2}$$