# Discontinuous Optical Flow: Recent Theoretical Results

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## Abstract

*We present a theory of occlusion in the context of optical flow computation. In this contribution we derive, under models of constant and linear optical flow, several propositions describing the frequency structure of motion discontinuities arising from occlusion events in the spatial domain. We show that a wealth of crucial information is most easily obtainable from the analysis of the structure of occlusion in Fourier space. For instance, the identification of the occluding and occluded velocities is possible. We also demonstrate the geometrical properties of degenerate cases occurring when signals suffer from the aperture problem. In particular, we show that the full velocity of a degenerate occluding signal is almost always obtainable at the occlusion. We conclude by showing that additive translucency phenomena may be reduced to special cases of the theory.*

## 1 Introduction

Traditionally, image motion and its approximation known as optical flow have been treated as continuous functions of the image domain [4]. However, in realistic imagery, one exceedingly rarely finds cases verifying this hypothesis. Many phenomena may cause discontinuities in the optical flow function of imagery [6]. Among them, occlusion and translucency are frequent causes of discontinuities in realistic imagery. In addition, their information content is useful to later stages of processing [2] such as motion segmentation [1] and 3-d surface reconstruction [7].

Occlusion boundaries are described as the partial occlusion of a surface by another, while translucency is defined as occlusion of a surface by translucent material. In realistic imagery, one finds occlusion to be the most frequent cause of discontinuous motion.

In this contribution, we investigate the structure of occlusion and transparency in the Fourier domain. Our motivation comes from the fact that the very presence of occlusions or surface translucence renders the usage of usual methods such as differentiation and region matching very difficult. Further, we postulate that spatial information constitutes an obstacle to determining discontinuous optical flow caused by occlusion. We derive, under models of constant and linear optical flow, several propositions describing the structure of motion discontinuities in Fourier space, arising from occlusion events in the spatial domain. We show that a wealth of crucial information is most easily obtainable from the analysis of the structure of occlusion in Fourier space. For instance, the identification of the occluding and occluded velocities is possible. We also demonstrate the geometrical properties of degenerate cases occurring when signals suffer from the aperture problem. In particular, we show that the full velocity of an occluding signal suffering from the aperture problem is almost always obtainable. We conclude by showing that additive translucency phenomena may be reduced to special cases of the theory.

## 2 Multiple Motions

Given an arbitrary environment and a moving visual sensor, the motion field generated onto the imaging plane by a 3-d scene within the visual field is represented as a function of the motion parameters of the visual sensor, usually expressed as instantaneous translation $\mathbf{T} = (T_x, T_y, T_z)^T$ and rotation $\mathbf{\Omega} = (\Omega_x, \Omega_y, \Omega_z)^T$:

$$\mathbf{v} = \begin{pmatrix} Z(\mathbf{x})^{-1}(xT_z - T_x) \\ Z(\mathbf{x})^{-1}(yT_z - T_y) \end{pmatrix} + \begin{pmatrix} xy\Omega_x - (1+x^2)\Omega_y + y\Omega_z \\ (1+y^2)\Omega_x - xy\Omega_y - x\Omega_z \end{pmatrix}, \quad (1)$$

where $\mathbf{x} = (x,y)^T$ is the perspective projection of a point $\mathbf{P}^T = (X, Y, Z)$ in the visual field. Assuming that the motion of the visual sensor is continuous (that is to say: $\mathbf{\Omega}$ and $\mathbf{T}$ are differentiable with respect to time), discontinuities in image motion are then introduced in (1) whenever the depth function $Z(\mathbf{x})$ is other than single-valued and differentiable. The occurrence of occlusion causes the depth function to exhibit a discontinuity, whereas translucency leads to a multiple-valued depth function.

Towards a useful generalization of this analysis, we show the structure of occlusion in Fourier space for one and two-dimensional signals and various models of optical flow. For instance, we consider constant and linear models of the optical flow function in both 1 and 2D. We also explore degenerate cases in which