Finding the Exact Optical Flow: a Maximum Flow Formulation

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Abstract

This paper introduces a new efficient method for finding the optical flow for a special type of motion and smoothness constrain. The optimisation is mapped into a maximum-flow problem in a graph, which is solved efficiently and leads to an optimal solution. Previously, similar methods were used to solve stereo correspondence [11, 9] and maximum *a posteriori* estimation problem in Markov random fields [2, 3, 6, 10].

After a general introduction to optical flow, we present a modified version of Horn and Schunck's method [7], then we present how we map the optical flow optimisation problem into a flow graph. Experimental results on simulated data are then provided and compared to the modified Horn and Schunck's method. They clearly show the potential of our formulation with normalised errors of 9.90% with our method vs. 19.87% with the Horn and Schunck's method. Both algorithms were also tested with realistic data in cineangiography of artery to assess blood flow. Again, our method shows a better velocity profile assessment (1.97% vs. 8.32% of error). These results were expected because the maximum-flow formulation is not iterative and leads to a global optimum.

1 Introduction

There is an abundant literature about the computation of the optical flow (see Barron *et al.* [1] for a summary of the major methods). In this paper, we focus our attention on the optimisation involved in a special gradient-based method and show how to map the optimisation into a flow problem in a graph. Similar works have been done in the field of stereo correspondence [3, 9, 11], image segmentation [10] and maximum *a posteriori* estimation in Markov random fields [2, 3, 6]. Before getting to our method, we introduce the basics of gradient-based optical flow calculation [1, 7], then we present a modified version of Horn and Schunck's algorithm [7] and finally, we proceed with our new method based on a maximum-flow formulation.

2 Optical flow

A 2-D sequence of two or more images is mathematically described as a function $I(i,t)$, where I is the image intensity at time t and at position $i = (x,y) \in \mathcal{S}$ and $\mathcal{S} = \{0,\ldots,m-1\}$ is the set of all pixels. By the chain rule of derivation, we obtain a formulation for the total rate of change of brightness

$$\frac{dI}{dt} = \frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} \tag{1}$$

where $\partial I/\partial x$, $\partial I/\partial y$ and $\partial I/\partial t$ can be computed directly from a pair of images $I(i,t)$ and $I(i,t+\Delta t)$ and will be abbreviated by I_x, I_y and I_t. It remains to determine the brightness change dI/dt and the x and y components of velocity, denoted by $u = dx/dt$ and $v = dy/dt$ respectively. To solve equation (1), for the velocities (u,v) and the brightness change dI/dt, additional constraints must be applied to restrict the allowable motions. Such constraints are constructed according to prior knowledge we have about the real motion and brightness change. Horn and Schunck [7] suppose the intensity is conserved, so the brightness change of a particular point in the image is equal to $dI/dt = 0$. Equation (1) is rewritten as

$$\frac{dI}{dt} = I_x u + I_y v + I_t = 0 \tag{2}$$

Because this is a single equation with two unknowns, Horn and Schunck [7, 1] added a smoothness constraint to solve the optical flow. We now proceed with our modified version of their algorithm.

2.1 Modified Horn and Schunck's method

In our applications, we set $v = 0$ because there is no motion along the y-axes. Equation (2) is reformulated as

$$I_{x_i} u_i + I_{t_i} = 0 \tag{3}$$