A General Markov Random Field Model by Partial Differential Equations Method

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Abstract

Markov Random Field(MRF) is efficient to model texture images, and many applicable models have been developed. The interpretations of these available models provide many different views of the image model structure and their application. In this paper, we present a new MRF model called PDE-MRF model based on constructing generic energy function by Partial Differential Equations(PDE) method. This model not only includes the majority of the present particular MRF models but also is a uniform template to develop some new models.

KEYWORDS: Markov Random Field (MRF), Texture Modeling, Partial Differential Equations (PDEs), PDE-MRF Model, Energy Function

1 Introduction

Markov Random Field(MRF) was introduced into image processing by J. Besag[1], and it has been applied to a lot of practical problems. Such fields of image processing as image restoration, image segmentation and image classification employ MRF model as one of their main tools. Among these applications, texture analysis, texture modeling are two important and fruitful fields.

Generally, texture generation process involves a stochastic assumption[2]. The gray level at a point in a texture image is highly dependent on the gray levels of neighboring points unless the image is simply random noise. MRF is able to describe this dependence. According to variable kinds of textures and applications, many particular MRF models have been presented, e.g., Auto Models(including Auto-Logistic model, Auto-Binomial model, Auto-Normal model and Auto-Poisson model)[1], Multi-Level Logistic Model(MLL)[3], Hierarchical GRF Model[4], and FRAME[5]. These models have different structures, and their parameters are determined not mainly by the texture types but by experiments of goodness-of-fit of the observed texture and the generated one. As a result, randomness is inconsistently added to the structure of the model and definition of the parameters.

Partial Differential Equation(PDE) methods have been applied to image representation for nearly forty years. A. Turing is the researcher who first used PDE model to synthesize animal patterns, in which the method is called as reaction-diffusion[6]. A general outline of relations between PDE and image processing was presented by A. K. Jain in his paper[7]. After introducing the notion scale-space into PDE methods by Koenderink[8] and Witkin[9], several PDE models for image processing have been created, for example, anisotropy-diffusion PDE models by Perona and Malik[10], anisotropy reaction-diffusion by Witkin and Kass[11] and so on. These models not only focus on image restoration or edge detection but also can be applied to texture analysis.

The reasons for many applications of PDE models to image processing are rooted in the advantages of PDEs. First, PDEs can satisfy such stability requirements for image processing as locality and causality. Second, the PDE formulation is natural in order to combine algorithms. Another important advantage of the PDE approach is the possibility of achieving high accuracy and stability aided by available numerical analysis.

It is noted that PDEs mainly possess deterministic property that is always corrupted by uncertainties in image data. Another problem is the inflexibility of choosing coefficients and equation forms. A realistic image model should integrate the known deterministic