Statistical comparison of images using Gibbs random fields

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Abstract

The statistical tools used to compare images are often based on multivariate statistics and use images as observations. This approach does not take into account the spatial and dependence structures in the images. Here we follow another approach in the sense that the number of pixels in the images is the number of observations. We also take into account spatial and dependence structures by modelling images with Gibbs random fields with nearest-neighbors interactions. The idea of the proposed method of comparison of images is to model the images by Gibbs random fields with a finite number of parameters and to test equality of parameters between the pooled images of each group. We obtain a test statistic that is easy to calculate, and the limiting distribution of the statistic is a chi-square distribution. Applications to handwrittens signatures and pattern recognition are presented.

1 Introduction

In many applications one wants to compare images of the same object. For example, one is interested in comparing fingerprints, handwritten signatures, radar images, etc.

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In what follows we assume that the images are comparable in the sense that they have the same size and the same orientations (e.g. [1]). The idea of the proposed method of comparison of images is to model the images by Gibbs random fields with a finite number of parameters and to test equality of parameters between the pooled images of each group.

In Section 2, images models with an arbitrary number of gray levels are introduced. Section 3 deals with methods for estimating the parameters of these images and an appropriate test statistic is defined. Section 4 is devoted to examples of applications for binary images, while the Appendix contains the proof of the asymptotic behavior of the proposed test statistic together with efficient Monte-Carlo Markov Chain methods for generating images.

2 Gibbs random fields

2.1 Notations

The set of pixels will be denoted by Λ. It is assumed that Λ is a rectangle of $\mathbb{Z}^2$. The "color" of the pixel at site x will be denoted by $\sigma(x) \in C \subset \{0, 1, \ldots\}$, $x \in \Lambda$, where a white pixel has value 0, and a black pixel has value 1. Set $\sigma_A = C^A = \{\sigma(x); x \in A\}$. For example, for black and white images or binary images, $C = \{0, 1\}$. For gray level images, $C = \{0, 1, \ldots g - 1\}$ and g is the number of gray levels. For sake of simplicity, extend every σ so that $\sigma(x) = 0$ whenever $x \notin \Lambda$.

If one wants to use statistics, one has to define an interesting parameterized family of distributions on the set of all possible images $\Omega = C^\Lambda$. The desirable features of random images are

- relatively few parameters;
- "Markov property": the law of the color of a pixel at site x given all other pixels should depend only on the colors of the pixels in a given neighborhood N_x of x;
- "stationarity": $N_x = x + N$, for any $x \in \Lambda$;