Estimating Expansion Rates from Range Data Sequences

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Abstract

We present a method to compute surface expansion rates from sequences of range data. Towards this end the 3D velocity field (range flow) is extracted first and then used in a second step to estimate the local area expansion. A detailed performance analysis is presented and the method is applied on two real examples.

1 Introduction

We denote the instantaneous velocity field that describes the motion of a deformable surface as *range flow*. The term range flow is used as we derive this velocity field from sequences of range data sets. Together with the 3D structure the range flow field can be used to study the dynamic changes of such surfaces. One interesting question is whether the surface area changes during the motion. This can for example be used to study growth processes in biological systems such as leaves or skin.

The same displacement vector field has also been called scene flow when computed directly from stereo image sequences [14, 16, 3]. We present range flow estimation in a differential framework that is related to optical flow algorithms [8, 1, 9, 10, 11]. Other approaches that use deformable models have been reported before [12, 15].

Paper organisation: In Sect. 2 we review the concept of range flow estimation. Then we define a formula for local expansion rates in Sect. 3, an error analysis by means of error propagation is presented in Sect. 3.1. Experiments are reported in Sect. 4, where we first conduct a detailed performance analysis on synthetic test data (Sect. 4.1). Finally we apply our technique on two real examples in Sect. 4.2.

2 Range Flow

The concept of range flow estimation is briefly reviewed in the following. For a more detailed description we refer to [9, 10, 1].

2.1 Constraint Equations

Note that a range sensor produces one data set for each of X,Y and Z on its grid ($X = X(x, y, t)$ etc.). Here sensor coordinates are denoted by (x,y). The three components of the range flow field are the total derivatives of the world coordinates with respect to time ($U = \frac{dX}{dt}$ etc.). This can be expressed in the following equations:

$$\begin{aligned} U &= \partial_x X\,\dot{x} + \partial_y X\,\dot{y} + \partial_t X\,, & (1)\\ V &= \partial_x Y\,\dot{x} + \partial_y Y\,\dot{y} + \partial_t Y\,, & (2)\\ W &= \partial_x Z\,\dot{x} + \partial_y Z\,\dot{y} + \partial_t Z\,. & (3) \end{aligned}$$

The total time derivative is indicated by a dot. As we are not interested in the rates of change on the sensor coordinate frame we eliminate $\dot{x}$ and $\dot{y}$ to obtain the range flow motion constraint expressed in sensor coordinates:

$$\frac{\partial(Z,Y)}{\partial(x,y)}U + \frac{\partial(X,Z)}{\partial(x,y)}V + \frac{\partial(Y,X)}{\partial(x,y)}W + \frac{\partial(X,Y,Z)}{\partial(x,y,t)} = 0\,, \quad (4)$$

where $\frac{\partial(Z,Y)}{\partial(x,y)}$ is the Jacobian of Z,Y with respect to x,y. Notice that the Jacobians are readily computed from the derivatives of X,Y,Z in the sensor frame obtained by convolving the data sets with derivative kernels. In the following special derivative kernels optimised for directional invariance are used [7]:

$$\begin{aligned} \partial_x &= (0.084,\ 0.332,\ 0,\ -0.332,\ -0.084)_x \\ &* \ (0.023,\ 0.242,\ 0.470,\ 0.242,\ 0.023)_y \\ &* \ (0.023,\ 0.242,\ 0.470,\ 0.242,\ 0.023)_t\,. \quad (5) \end{aligned}$$

The other derivatives are computed in the same manner. In practice many sensors have aligned world and sensor coordinate systems which implies $\partial_y X = \partial_x Y = 0$. Yet Eq. (4) poses the general constraint independent of a particular sensor.

The usage of intensity data in addition to the range data can improve both the accuracy and density of the estimated range flow significantly [9]. We assume that the intensity